Minimum wages, wage dispersion and financial constraints in firms[†]

Hamzeh Arabzadeh[1], Almut Balleer[1,2], Britta Gehrke[3,4,*], and Ahmet Ali Taskin[4]

[1] *RWTH Aachen University, Germany*
[2] *Institute for International Economic Studies, Sweden, and CEPR*
[3] *Freie University Berlin, Germany, and IZA*
[4] *Institute for Employment Research (IAB), Germany*

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Abstract

This paper studies how minimum wages affect the wage distribution if firms face financial constraints. Using German employer-employee data and firm balance sheets, we document that the within-firm wage dispersion decreases more with higher minimum wages when firms are financially constrained. We introduce financial frictions into a search and matching labor market model with stochastic job matching, imperfect information, and endogenous effort. In line with the empirical literature, the model predicts that a higher minimum wage reduces hirings and separations. Firms become more selective such that their employment and wage dispersion fall. If effort increases strongly, firms may increase employment at the expense of higher wage dispersion. Financially constrained firms are more selective and reward effort less. As a result, within-firm wage dispersion and employment in these firms fall more with the minimum wage.

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[*] Corresponding author: Freie Universität Berlin, School of Business and Economics, Boltzmannstr. 20, 14195 Berlin, Germany, Email: *britta.gehrke@fu-berlin.de*.

1 Introduction

With relatively low unemployment in Europe compared to historical levels, distributional effects of labor market policies have come to the forefront of the economic debate. This paper considers the distributional implications of the minimum wage. Policymakers in many countries including the US, the UK, and the EU are considering or implementing minimum wage increases. In the accompanying debate, potential benefits such as a reduction in wage inequality or higher productivity and wages are confronted with potential costs such as adverse employment effects. Several open questions remain in this debate.[1] It is unclear, for example, to what extent the existing literature can speak to an economic environment where financial constraints become binding, e.g., in a recession or a financial crisis.

Previous literature has demonstrated that minimum wages can reduce wage inequality (Bossler and Schank, 2023, Engbom and Moser, 2022, Autor et al., 2016). This happens partly by construction as a minimum wage cuts the lower tail of the wage distribution. The overall effects on wage inequality due to firms' hiring and firing adjustment, changes in labor market conditions and wage spillovers on workers not directly affected by the minimum wage are less clear, however. Firm-specific aspects may interact with all of these dimensions rendering the effect of the minimum wage heterogeneous across firms. When firms rely on external finance, minimum wages intensify the need for external finance as they raise firms' costs. In this paper, we document that the effect of minimum wages on wage dispersion indeed differs by firms' financial conditions. We rationalize this finding with a labor market model that illustrates the mechanisms behind our empirical result.

To set the stage, we present novel stylized facts on the relationship between minimum wages, wage dispersion and firms' financial constraints. We use the introduction of a federal minimum wage in Germany as a quasi-natural experiment. This policy took effect on January 1, 2015 and required an hourly gross wage of at least 8.50 Euros. This directly affected about 15 percent of German workers as they earned below the minimum wage limit before (Dustmann et al., 2022). We separately analyze in-firm wage dispersion outcomes by firms' financial constraints.[2] For this purpose, we combine two datasets: administrative employment biographies and wages for the population of German workers and annual financial accounts for German firms. We report

[1] See for example the recent overview by Manning (2021).

[2] While within-firm wage dispersion does not translate one-to-one to wage inequality, it contributes the major share. For example, Song et al. (2019) find that within-firm wage dispersion contributes between $60-70\%$ percent (between-firm dispersion roughly $30-40\%$) to US wage dispersion.

baseline results for wage dispersion between the 90th and 10th percentile and use leverage to measure financial constraints. We document a clear reduction in within-firm wage dispersion in 2015 that is stronger for firms that are more affected by the minimum wage. For these firms, we find a decline of 1.8 percentage points. For financially constrained firms, the reduction is more pronounced (and statistically significantly different) with an additional decline of up to 1.6 percentage points. Our results are not driven by our choice of measure for financial constraints or by constrained firms being less productive or having grown too much prior to the reform.

Based on these empirical findings, we propose a theoretical labor market model that takes into account how firms' labor demand and financial conditions, the wage bargaining and labor search interact with the minimum wage. The model features two main channels: the selection and the effort channel. We follow Brochu and Green (2013) and combine a search and matching model with heterogeneous match productivity and stochastic job matching (Pissarides (2000), chapter 2) with imperfect information about true match-productivity at the time of hiring, but perfect information after training. Higher minimum wages increase the reservation productivity, i.e., firms become more selective. If observed and true match-productivity are correlated and the training cost is indexed to the minimum wage, higher minimum wages induce firms to hire more productive workers on average and, therefore, fire less after training.[3] This model then predicts less firing with the minimum wage, but hiring falls by more such that employment and wage dispersion unambiguously decrease. This is the "*selection effect*" of the minimum wage.

Although the selection effect per se explains our empirical finding on wage dispersion, it is at odds with empirical evidence that questions strong negative employment effects of the minimum wage. To speak to this empirical fact, we introduce an on-the-job effort channel. Effort increases match output and follows the empirical evidence of Coviello et al. (2022) who show, using US data, that minimum wages can raise individual worker productivity. A rise in the minimum wage then induces low-productivity workers to exert more effort to keep their jobs, reducing the firing rate more than without endogenous effort. We refer to this as the "*effort effect*" of the minimum wage. If the effort effect reduces separations enough, employment may rise with a higher minimum wage.[4] The effort effect may increase wage dispersion if employment rises

[3] An alternative modeling strategy to capture that separations fall with minimum wages is job-to-job transitions similar to Burdett and Mortensen (1998). Then, separations take the form of quits (see, for example, Dube et al. (2016); Portugal and Cardoso (2006)). Here, in line with empirical evidence from Brochu and Green (2013), the major decline in separation takes the form of layoffs.

[4] An alternative explanation for positive employment effects of a minimum wage is labor market monopsony power. See e.g., Azar et al. (2023).

with the minimum wage. This then creates a positive spillover on market wages as the outside option of workers in the bargaining (their reservation wage) improves.

Given our interest in financial constraints, we incorporate financial frictions due to costly-state verification as in Carlstrom and Fuerst (1998) in our model. Due to a cash flow mismatch between paying wages and the realization of sales firms must finance wages externally. The conditions of these loans are determined through a financial contract between firms and lenders (banks). If wages are externally financed, the reservation productivities for hiring and firing increase which leads to lower wage dispersion. In other words, financial constraints intensify the selection effect of a minimum wage. The effort effect, however, is weakened by financial constraints. Financial constraints introduce a financial labor wedge between output-per-worker and wages, implying that financially constrained firms pay on average lower wages. A binding minimum wage raises wage costs and hence the need for external finance and financial constraints. As wages are lower for a given output, workers in these firms exert lower effort. As a result, financial constraints imply a stronger reduction in wage dispersion also through the lens of the effort channel (less of a buffering). The effect of financial constraints on wage dispersion through both channels is thus always negative and in line with our empirical evidence.

We derive these results formally using comparative statics. However, the overall effect on wage dispersion and employment in the model depends on the strength of the selection vis-à-vis the effort channel and is hence a quantitative question. We calibrate the model to the German economy when it introduced the minimum wage in 2015. We find that our model can replicate a zero employment effect of the minimum wage with a negative effect on wage dispersion. This effect increases with financial constraints and is quantitatively in line with the data. We further explore the role of the effort versus the selection channel in counterfactual exercises with different effort levels. We document a clear trade-off between the minimum wage effect on employment and wage dispersion.

Our results demonstrate that firms' financial constraints matter for how firms react to a minimum wage and that financial constraints can therefore be behind heterogeneous distributional effects of minimum wage policies. Our results further suggest that minimum wage policies can have state-dependent effects, i.e., different effects of minimum wage changes in financial recessions or booms compared to regular recessions or booms. Nevertheless, our analysis abstracts from potential general equilibrium effects of minimum wages and financial constraints (e.g., on prices, consumption), but focuses on the interplay of the labor and the financial market.

To the best of our knowledge, this paper is the first to study the interaction of financial frictions with the minimum wage. Our paper is related to Berger et al. (2022), Drechsel-Grau (2022) and Di Nola et al. (2023) who investigate minimum wages in structural macroeconomic models with frictions, but none of these look into financial constraints. The empirical effect of minimum wages on firms is documented in a large and fast-growing literature. It is well-established that higher minimum wages reduce both separation and hiring rates (Coviello et al., 2022; Dube et al., 2016; Brochu and Green, 2013) and the number of posted vacancies (Kudlyak et al., 2022). The empirical evidence on the effect of a minimum wage on employment and on output-per-worker is mixed. Several studies, including Card (1992a,b) and Card and Krueger (1994) point to instances where an increase in the minimum wage resulted in increased employment of low-wage workers. Others find evidence that the minimum wage has insignificant or even negative effects on the employment of low-paid workers (e.g., Brochu and Green, 2013; Dube et al., 2016; Jardim et al., 2018; Yuen, 2003).[5] For output per worker, Ku (2022) and Hill (2018) find opposing results. Engbom and Moser (2022), Dustmann et al. (2022), Bossler and Schank (2023), or Cengiz et al. (2019) document positive effects on productivity and employment of a minimum wage due to reallocation of workers to higher-paying and more productive firms. Di Nola et al. (2023) show that a minimum wage triggers reallocation of workers from marginal employment to jobs with longer hours, in particular for women. Link (2022) documents that firms trade off cuts in employment with higher prices. Complementary to our study, Adamopoulou et al. (2021) show that minimum wages alter the allocation of firm-idiosyncratic risk across workers.

The rest of the paper is organized as follows. Section 2 presents stylized facts on financial constraints, wage dispersion and minimum wages, Section 3 outlines the model and the most important mechanisms. In Section 4, we discuss the minimum wage effects based on a quantitative model. Finally, Section 5 concludes.

2 Stylized facts

In this section, we present novel evidence on the effect of financial constraints and minimum wages on firm-level wage dispersion. We use the introduction of a federal minimum wage in

[5] Neumark et al. (2007) provide a survey of the empirical findings of studies on the relationship between minimum wages and employment. They conclude that among "the 33 studies judged the most credible, 28, or 85%, pointed to negative employment effects. [...] In contrast, few -if any- studies provide convincing evidence of positive employment effects of minimum wages."

Germany as a quasi-natural experiment. This policy took effect on January 1, 2015 and required an hourly gross wage of at least 8.50 Euros.[6] This had direct and indirect consequences for up to a quarter of German employees as they earned below or around the minimum wage ex-ante. It affected mainly immigrants, females, low-skilled and young workers, workers residing in East Germany, and workers in small establishments in particular in transportation, accommodation and food services (Dustmann et al., 2022). Different studies have evaluated the effects of this policy. For example, Dustmann et al. (2022) document that the minimum wage introduction raised wages, but did not reduce employment. Bossler and Schank (2023) show that it contributed substantially to a reduction in wage inequality. Complementary to these studies, we analyze the link between wage dispersion and firms' financial constraints.

2.1 Data

For this purpose, we combine three data sets: Our main data is the IAB Integrated Employment Biographies (IEB) which is employee-level administrative data of all jobs covered by social security (excluding civil servants and the self-employed). This data includes daily gross wages (base wage plus extra pay) for each job of a worker in a given establishment. The administrative data is characterized by detailed information on workers and establishments and a high degree of reliability of the earnings data since social security institutions run plausibility checks and sanction misreporting. However, the IEB data does not have information on hours worked. For this reason, we combine it with the administrative hours data as prepared by Vom Berge et al. (2023) which provides job-level hours worked information between 2011 and 2014.[7] Our third dataset is Bureau van Dijk's Dafne which compiles yearly financial accounts for public and private firms in Germany and allows us to measure firms' financial strength. Dafne provides balance sheet information (e.g., total assets, long and short-term debt, cash holdings etc.) for a large set of firms and income statements that include sales, profits, interest expenses etc. for larger firms only. We use a recent record linkage that allows to merge the establishments in the IAB data with firms in Dafne.[8]

[6]Before 2015, minimum wages existed in certain industries. The law on the nationwide minimum wage was passed by the German parliament in July 2014. Dustmann et al. (2022) provide an excellent discussion of the minimum wage introduction in Germany and the macroeconomic surroundings of the policy change. For comparison, with a minimum wage to median ratio of 0.48 according to OECD Statistics, the 2015 German minimum wage was in a similar order of magnitude as the UK policy, more binding compared to the US federal minimum wage, but less binding compared to the French policy.

[7]Vom Berge et al. (2023) develop a correction procedure that ensures that the hours data uniformly reflects contractual working hours.

[8]The data has been merged using record key linkage by the Research Data Center (FDZ) of the IAB using the firm name, legal form and address. This is an ongoing project that continuously updates and extends the

We follow Giroud and Mueller (2017) and use leverage as defined by the sum of total debt in the form of current and long-term liabilities to total assets as a measure of financial constraints. We compute leverage as an average for the years between 2011 and 2014. We drop financial firms and firms in public administration and defense. We classify a firm as financially constrained if it has a leverage ratio higher than 50 percent.[9] We choose leverage as our preferred measure of financial constraints due to its wide availability and ease of interpretation. As robustness, we include firm controls to address alternative channels through which leverage might influence wage dispersion beyond financial constraints. Furthermore, we introduce an alternative financial constraint measure based on commonly used firm characteristics.

We then compile the universe of employees who have worked in those firms between 2012 and 2018. We remove trainees and apprentices and create a sample of unique employee-employer-year observations at June 30 of each year. We impute wages following Gartner et al. (2005) because the earnings information is top-coded in the social security records.

We use this sample to create a continuous measure of minimum wage affectedness ('bite') at the firm level pre-2015. Following an established approach in the minimum wage literature (e.g., Dustmann et al., 2022), we measure the firm's ex-ante exposure to the minimum wage as

$$bite_{ft} = \frac{\sum_{i \in f} hours_{ift} \max\{0, minwage - wage_{ift}\}}{\sum_{i \in f} hours_{ift} wage_{ift}}. \tag{1}$$

Here, $hours_{ift}$ denotes the daily hours worked of individual i in firm f at time t. $wage_{ift}$ refers to workers' hourly wages. This measure reflects the relative increase in the wage bill that a firm needs to bring all workers to the minimum wage. A high bite indicates that a firm faces a substantial increase in personnel expenses after the minimum wage introduction if they employ a large share of workers below the minimum wage and/or if they employ workers with wages far below the minimum wage. We calculate the average between 2012 and 2014 to take care of anticipation of the policy change.

To demonstrate the effect of the introduction of minimum wage in Germany on within-firm wage dispersion, we restrict our attention to firms that have at least four full-time employees before and after the policy. Following the literature, we use log differences between the 90th to 10th and the 75th to 25th wage percentiles as measures for wage dispersion. Depending on the

existing record linkages, see e.g., Antoni et al. (2018). The data has among others been used in the recent study by Jäger et al. (2021).

[9] This assigns almost 30 percent of our firm sample to be financially constrained.

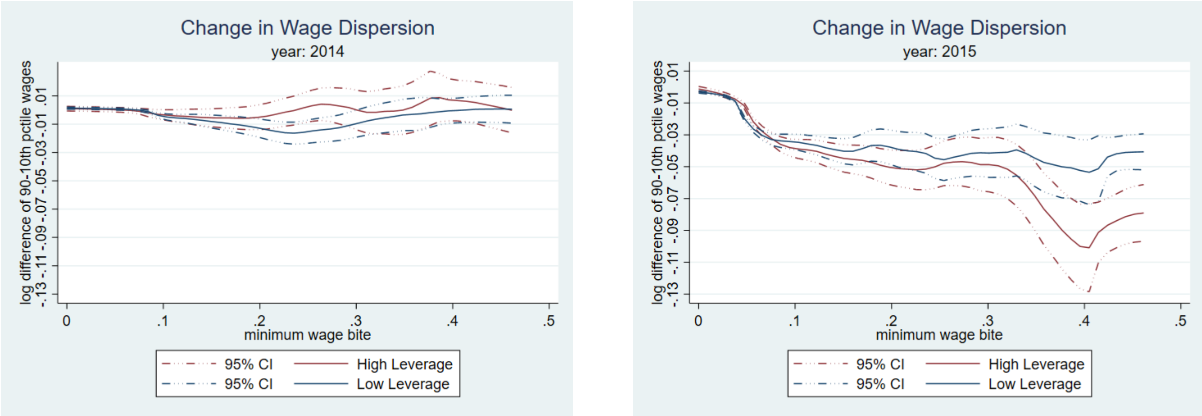

Figure 1: Changes in 90th-to-10th wage percentiles for high vs. low leverage firms with respect to average minimum wage bite before the introduction of statuary minimum wage in 2015. Source: IEB, Dafne and authors' calculations.

minimum wage bite, the lower end of the wage dispersion measure may therefore include or not include minimum wage takers.

2.2 Descriptive statistics

Figure 1 documents the change in wage dispersion by minimum wage bite and by financial constraints. The right panel of Figure 1 shows a reduction in wage dispersion in 2015 that is stronger, the more affected a firm is by the minimum wage. The reduction in wage dispersion is even stronger for financially constrained firms (red lines). In contrast, the left panel of Figure 1 presents a placebo test: In 2014 before the introduction of the minimum wage, there were no relevant differences in wage dispersion by minimum wage bite or by financial constraints.

Next, we document summary statistics of the wage distribution within a 3-year window before and after the introduction of minimum wage. We split the sample into three separate groups of firms: A control group with minimum wage bite below 0.05, a partial treatment group with minimum wage bite between 0.05 and 0.15, and a full treatment group with minimum wage bite above 0.15. This leaves a similar number of observations for treated and partially treated firms. Table 1 provides the within-firm wage distribution and wage dispersion before and after the minimum wage policy, separately for exposure to the minimum wage and separately by financial constraints.

We observe that firms that are unaffected by the minimum wage (control group with bite below 0.05) have, on average, higher wages and wage dispersion compared to high minimum wage bite firms. Although high-leverage firms exhibit (on average) lower wages, this does not translate into larger wage dispersion for any group. This holds irrespective of whether

	Control (minwage bite<=0.05)		Partial treatment (0.05<minwage bite<=0.15)		Treatment (minwage bite>0.15)	
	Low leverage	High leverage	Low leverage	High leverage	Low leverage	High leverage
Pre-policy (Time period: 2012-2014)						
$log(w_{10})$	4.3575	4.2007	3.8439	3.8307	3.7285	3.6709
$log(w_{25})$	4.4964	4.3418	4.0159	4.0104	3.8902	3.8458
$log(w_{50})$	4.6534	4.4876	4.1727	4.1645	4.0412	3.9999
$log(w_{75})$	4.8552	4.6661	4.3538	4.3357	4.2112	4.1666
$log(w_{90})$	5.1082	4.8909	4.5763	4.5434	4.4209	4.3649
$log(w_{90}) - log(w_{10})$	0.7507	0.6901	0.7324	0.7127	0.6924	0.6941
$log(w_{75}) - log(w_{25})$	0.3588	0.3243	0.3379	0.3254	0.3210	0.3208
Observations	446,096	162,617	67,640	33,189	46,829	25,589
Difference post (2015-2017)- vs. pre-policy (2012-2014)						
$log(w_{10})$	0.0460	0.0480	0.1127	0.1139	0.1530	0.1659
$log(w_{25})$	0.0437	0.0427	0.0865	0.0875	0.1348	0.1468
$log(w_{50})$	0.0446	0.0425	0.0734	0.0724	0.1174	0.1262
$log(w_{75})$	0.0454	0.0442	0.0686	0.0676	0.1040	0.1108
$log(w_{90})$	0.0420	0.0454	0.0674	0.0666	0.0990	0.1061
$log(w_{90}) - log(w_{10})$	-0.0040	-0.0026	-0.0453	-0.0473	-0.0540	-0.0598
$log(w_{75}) - log(w_{25})$	0.0017	0.0015	-0.0180	-0.0200	-0.0308	-0.0360

Table 1: Summary statistics on firm-level wages w and wage dispersion pre- and post-minimum wage. Wages are deflated by consumer price indices. Wage dispersion is defined as the log difference between the 90th to 10th and 75th to 25th wage percentiles at the firm level. High-leverage firms are those whose average debt-to-assets ratio before 2015 is above 50 percent. Minimum wage bite is calculated at the firm level using Equation (1). We classify firms in the treatment group whose minimum wage bite is above 0.15, in the partial treatment group whose minimum wage bite is between 0.05 and 0.15 and in the control group if minimum wage bite is below 0.05. Source: IEB, Dafne and authors' calculations.

we measure wage dispersion by the 90-to-10th percentile or the 75-to-25th percentile. After the introduction of the minimum wage, wages increased across the whole wage distribution. However, the increase is stronger at the low end of the distribution, i.e., for those that are more directly affected by the minimum wage. This implies that the higher the minimum wage bite the larger the reduction in wage dispersion. This pattern is reinforced by financial constraints: high-leverage firms in the treated group experience a 16.6 (10.6) percent increase at the 10th (90th) wage percentile translating into a 6.0 percentage points decline in wage dispersion after the introduction of minimum wage. Without financial constraints, wage dispersion falls by 5.4 percentage points. Unaffected firms experience a much smaller rise in wages across the distribution and wage dispersion remains basically unchanged.

We further analyze the effect of the German minimum wage introduction on firm productivity and employment (see Table A.1 in the Appendix). We find that high-leverage firms with a strong minimum wage bite (treated firms) reduce employment by more than 7 percent while their worker productivity rises by 0.8 percent (measured by worker-fixed effects in spirit of Abowd et al., 1999). The effects follow the same pattern for low-leverage firms but are quantitatively smaller (-2 and +0.6 percent for employment and productivity). In the partial treatment group, only the high-leverage firms reduced their employment (by 1.5 percent), whereas low-leverage firms increased their employment slightly (0.9 percent). The latter is also true for firms with no minimum wage bite (control firms). While firms in the partial treatment group change their average worker productivity only a little, firms in the control group experience a larger reduction.

2.3 Regression analysis

The decline in wage dispersion that we document above could result from mean reversion or other industry-location-specific macroeconomic factors unrelated to the minimum wage. Due to mean reversion in wages, it is possible that workers who earn a low wage experience a higher wage growth than workers who earn a high wage. Since high minimum wage bite firms employ a large share of low-wage workers, the trend in reversion could then induce a stronger decline in wage dispersion for those firms. The fact that high-leverage firms have, on average, lower wages would imply further downward pressure on wage dispersion. Moreover, possible trends in industry-location-specific wage inequality could bias our findings to the extent that they are correlated with exposure to the minimum wage and financial constraints.

To address these issues, we introduce a formal econometric model that investigates the role of financial constraints and minimum wage bite for wage dispersion Y_{ft} in firm f in year t after the introduction of the minimum wage

$$\begin{aligned} Y_{ft} = \alpha_f + \alpha_{rj} \ \times Post_t + \delta_{Post,1} bite_f \times Post_t + \delta_{Post,2} \times leverage_f \times Post_t \\ + \delta_{Post,3} bite_f \times leverage_f \times Post_t + \epsilon_{ft}. \end{aligned} \quad (2)$$

As discussed above, we use log differences between the 90th to 10th and the 75th to 25th wage percentiles as measures for wage dispersion. Our estimation strategy includes a triple interaction of minimum wage bite, $bite_f$, average leverage, $leverage_f$, and an indicator that denotes the time period after the introduction of the minimum wage, $Post_t$.[10] To control for industry-location-specific macroeconomic trends, we interact the policy indicator with region

[10] The firm-level variables minimum wage bite and leverage are calculated as pre-policy averages. The pre (post)-policy period is the years between 2012-2014 (2015-2017).

Wage dispersion	75th-to-25th-percentile			90th-to-10th-percentile		
	(1)	(2)	(3)	(4)	(5)	(6)
Minimum wage bite × post-2014	-0.0864***	-0.0598***	-0.0232	-0.1408***	-0.0886***	-0.0534
	(0.0072)	(0.0054)	(0.0227)	(0.0117)	(0.0087)	(0.0382)
Leverage × post-2014	-0.0012	0.0007	0.0019*	0.0026	0.0042**	0.0080***
	(0.0017)	(0.001)	(0.0011)	(0.0029)	(0.0017)	(0.002)
Minimum bite × leverage × post-2014	-0.0292**	-0.0333***	-0.0337***	-0.0514**	-0.0792***	-0.0945***
	(0.0145)	(0.011)	(0.0127)	(0.0244)	(0.0188)	(0.0224)
Fixed effects	No	Yes	Yes	No	Yes	Yes
Firm controls	No	No	Yes	No	No	Yes
Observations	1,501,901	1,487,404	1,091,446	1,501,901	1,487,404	1,091,446

Table 2: Effect of financial constraints on wage dispersion from estimating Equation (2). The dependent variable is wage dispersion defined as the log difference of the respective wage percentiles. Minimum wage bite is defined as in Equation (1), leverage is total debt-to-assets. We calculate pre-policy averages at the firm level. Pre (Post) policy time period includes years 2012-2014 (2015-2017). Fixed effects include time-invariant firm-fixed effects and a triple interaction of county-1-digit-industry-fixed effects with a policy indicator. Firm controls include triple interactions of pre-policy average total assets, firm age as of 2014, 3-year total assets growth between 2011 and 2014 and firm-level AKM fixed effects with minimum wage bite and the policy indicator. Standard errors (in parentheses) are clustered at the firm level. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

(county) r and (one digit) industry j fixed effects. Finally, we control for firm-fixed effects, α_f. Our parameter of interest, $\delta_{Post,3}$, identifies the additional effect of leverage on wage dispersion for a given minimum wage bite after the introduction of the minimum wage.

Our triple interaction estimation strategy allows us to identify the effect of financial constraints and minimum wages with relatively weak common trend assumptions. Olden and Møen (2022) demonstrate that for an unbiased triple difference estimator one needs only one parallel trend assumption, in ratios, to hold. For our purposes, this means that the relationship between wage dispersion and leverage could exhibit trends before introducing the minimum wage. Our parameter of interest, $\delta_{Post,3}$, is still unbiased as long as this pre-trend does not change with minimum wage bite. Moreover, we control for time-invariant firm factors and local-industry wage dispersion which further address violations of common trends. Nonetheless, we further check our results with a dynamic triple interaction model to check bite and leverage specific trends in wage dispersion prior to the introduction of the minimum wage.

The results in Table 2 support the descriptive observations. First, firms with a higher minimum wage bite experience a statistically significant reduction in wage dispersion with the introduction of the minimum wage. The estimation with firm and industry-location-policy fixed effects (column 5) shows that a firm with a 20 percent minimum wage bite experiences a decline in wage dispersion in terms of the 90th-to-10th percentile of 1.8 percentage points (0.2*0.0886). Second, this reduction becomes larger if the firms face financial constraints (statistically significant). With a minimum wage bite of 20 percent and median leverage (20.17%), there is an additional decline in wage dispersion between the 90th-to-10th percentile of 0.3 percentage points. For very high leverage (100%), this additional effect increases to 1.6 percentage points. The size of the effect confirms the earlier finding in Figure 1. If we measure wage dispersion by the 75th-to-25th percentile (columns 1-3), the effects become smaller but remain statistically significant. We conclude that financial constraints play an important role for the effect of the minimum wage on wage dispersion, with stronger effects in the tails.[11]

2.4 Robustness

Potential confounding mechanisms could compromise leverage as a measure of financial constraints. For example, Giroud and Mueller (2017) argue that low-productivity firms or firms that experience strong growth accumulate debt. This could imply that firms with high leverage experience a decline in wage dispersion after the introduction of the minimum wage not because of financial constraints but because they are less productive or have become larger in previous years. Moreover, existing literature has shown that age and size could be related to within-firm wage dispersion and can predict reliance on external finance (Mueller et al., 2017). To address these concerns, we add average total assets, age of the firm (the difference between the year 2014 and the year that the firm was incorporated), 3-year total assets growth between 2011 and 2014, and firm-level productivity (firm-fixed effects in the spirit of Abowd et al., 1999) as controls and as triple interactions with minimum wage bite and the policy indicator. Columns 3 and 6 in Table 2 report the results of these regressions. Most importantly, the triple interaction term of minimum wage bite, leverage and the post-policy indicator remains negative and significant and becomes even larger (in absolute terms). We conclude that, as in Giroud and Mueller (2017), leverage indeed acts as a good proxy for financial constraints and firms with high leverage experience a further decline in wage dispersion beyond alternative channels stemming from size, age, growth and productivity.[12]

[11] If we run separate regressions for the lower and upper wage percentiles, we find that (1) wages generally increase with the introduction of the minimum wage, but more in lower percentiles compared to upper percentiles, (2) higher leverage in firms is associated with lower wages, (3) high-leverage firms increase wages by more with the minimum wage, but this effect gradually disappears in higher percentiles. See Appendix Table A.2.

[12] For readability, we do not discuss the effects of the control variables in detail. One interesting observation is that firm productivity interacts with the effect of minimum wages on wage dispersion. Firms with high productivity experienced a lower decrease in wage dispersion compared to low-productivity firms after the introduction

Next, we address heterogeneous mean reversion as a confounding effect. We replace the regression model above with a dynamic triple interaction model. There, we interact year-fixed effects with pre-policy minimum wage bite and leverage. The results in Appendix Table A.3 do not fully rule out mean reversion for high minimum wage firms: we observe a slight decline in wage dispersion before the introduction of the minimum wage, but this decline becomes much larger afterwards (columns 1 and 3). Leverage alone and its interaction with minimum wage bite did not exhibit any discernible trend before the introduction of the minimum wage. Afterwards, high-leverage firms with high minimum wage bite experience a significant fall in wage dispersion. We conclude that mean reversion does not play any role in the observed effect of financial constraints.

Finally, we replace leverage with an alternative measure of financial constraints, in particular, an index that we develop in line with the finance literature. Popular measures are the Kaplan and Zingales (1997) (KZ) and the Whited and Wu (2006) (WW) index, both of which combine balance sheet information in an index for financial constraints. However, as argued by Schauer et al. (2019), these measures are applicable almost exclusively to public firms (e.g., data on dividend payments are not available for private firms). Instead, we focus on variables that are available for a large set of firms including private firms. Following Mulier et al. (2016), we create an indicator based on the firm's average size, age, average cash holdings to total assets and average leverage.[13] We create the index for the period before the introduction of the minimum wage and apply a score of one if the firm is smaller, younger, more leveraged or has a lower cash ratio than its peers within the same county and industry. The firm is labeled as financially constrained if the index has at least a score of three based on these characteristics. We repeat the same regression as in Equation (2) with this alternative financial constraint indicator. The results in Appendix Table A.4 confirm our earlier findings: Firms with financial constraints experienced a larger decline in wage dispersion with the minimum wage.

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of the minimum wage.

[13] Leverage is also used in the KZ and WW index and by Mulier et al. (2016). Size and age are commonly suggested indicators of financial constraints as in Whited and Wu (2006) and Hadlock and Pierce (2010) for public firms and in Mulier et al. (2016) and Schauer et al. (2019) for private firms. Cash holdings also appear in multiple indices, e.g., in the KZ index and in Schauer et al. (2019).

3 The model

Next, we propose a theoretical model that provides mechanisms to explain why wage dispersion falls more in firms with financial constraints. Our model focuses on the labor market. In this sense, it is partial equilibrium as we do not model the goods market or a government budget. We start with describing the general setting and the labor market (Section 3.1), then we discuss the financial market and the financial friction (Section 3.2). Sections 3.3 and 3.4 lay out the firm problem and wages in detail. We then use the model to describe the steady-state equilibrium and workers' effort choice (Section 3.5) and comparative statics (Section 3.6).

3.1 General setting and the labor market

We study an economy where infinitely many identical workers indexed by i and firms indexed by j meet with search and matching frictions. Workers and firms are infinitely-lived and of unit mass. Time is discrete. Each period, firms post vacancies at a convex cost $C_v(v)$, with $C_v(0) > 0$, $C_v'(.) > 0$ and $C_v''(.) > 0$. Unemployed workers and firms meet according to a constant return to scale matching technology, $\mathcal{M}(v,u) = \xi u^{\varepsilon} v^{1-\varepsilon}$, where $u = 1 - n$ is the unemployment rate, n is the employment rate, v denotes vacancies and $\theta(:= \frac{v}{u})$ is labor market tightness. Consequently, the probability that a job seeker meets an employer is $f(\theta) := \frac{\mathcal{M}(v,u)}{u}$ and the probability that a vacancy meets a job seeker is $p(\theta) := \frac{\mathcal{M}(v,u)}{v}$.

While workers and firms are homogeneous, job matches differ in their match-specific productivity x_{ij}. Upon hiring, firms observe only an erroneous measure of match productivity, z_{ij} (hereinafter, "observable productivity"), such that $x_{ij} = z_{ij} + \epsilon_{ij}$. The productivity draws are random and independent with $z_{ij} \sim \mathcal{N}(\mu_z, \sigma_z^2)$, $\epsilon_{ij} \sim \mathcal{N}(0,1)$, and hence, $x_{ij} \sim \mathcal{N}(\mu_z, \sigma_x^2)$ with $\sigma_x^2 = \sigma_z^2 + 1$. Firms have to decide whether to hire based on observable productivity z_{ij}. Only after training, the true productivity x_{ij} of the match is revealed and workers become productive. Match-specific productivity x_{ij} stays constant as long as the match exists. This setting follows Brochu and Green (2013) and introduces a wedge between hiring and firing.

Worker i decides how much effort e_{ij} to exert on the job subject to cost of effort $C_e(e_{ij}) = c_e e_{ij}^2$. Match-specific output is then $y_{ij} = x_{ij} + e_{ij}$. When true match-productivity is revealed, the firm decides whether to fire the worker. Our setup therefore implies two productivity thresholds (i) a *hiring cutoff* (z_j^*) and (ii) a *firing cutoff* (x_j^*). Pissarides (2000) calls the latter "*reservation productivity*". Firms hire a worker if $z_{ij} \geq z_j^*$, and they keep the hired worker after training if $x_{ij} \geq x_j^*$. In fact, firms fire based on reservation output y_j^*. Conditional on workers' effort choice, this then determines the productivity threshold x_j^*.

Firms are risk-neutral and produce with a linear production function in labor input. Total firm output in period t is then given by

$$y_{jt} = \int_i y_{ijt} di = \int_i (x_{ijt} + e_{ijt}) di. \tag{3}$$

In every period, a share δ of matches is exogenously separated. This setting implies the following law of motion for employment n_{jt} in firm j

$$n_{jt+1} = (1-\delta) n_{jt} + v_{jt} p(\theta_t) \big(1 - H_{jt}^x(x_{jt}^*)\big), \tag{4}$$

where $1 - H_{jt}^x(x_{jt}^*)$ is the probability that a worker who meets the firm is hired and becomes active in the next period and is not fired. The productivity distribution $H_{jt}^x(.)$ is endogenous and depends on the cumulative distribution functions of z_{ij} and ϵ_{ij}, $H^z(.)$ and $H^\epsilon(.)$. This probability conditions on the worker having been hired, i.e., we integrate over all erroneous productivity realizations $z_{ij} \geq z_j^*$. As a result, $1 - H_{jt}^x(x_{jt}^*)$ is given by

$$1 - H_{jt}^x(x_{jt}^*) = \int_{z_{jt}^*}^{\infty} \Big[\int_{\epsilon_{jt} = x_{jt}^* - z_{ijt}}^{\infty} h^\epsilon(\epsilon_{ijt}) d\epsilon \Big] h^z(z_{ijt}) dz = \int_{z_{jt}^*}^{\infty} \big(1 - H^\epsilon(x_{jt}^* - z_{ijt})\big) h^z(z_{ijt}) dz. \tag{5}$$

Wages for productive workers are set by Nash wage bargaining, but the government may introduce a lower wage bound, i.e., a minimum wage. Firms pay training costs $c_\tau w^m$ during training, where $c_\tau > 0$ is a parameter and w^m is the minimum wage.[14] Firms need external finance to pay for wage costs. Workers receive market wages or the minimum wage depending on their draw of match-specific productivity plus effort. If unemployed, workers receive unemployment benefits b.

Firms have many workers with different wages (depending on minimum wage regulation, match-specific productivity, and workers' effort). Wage dispersion is the difference between various percentiles of this within-firm wage distribution which we will compare to the empirical results in Section 2.

[14] With zero training costs, firms hire all workers that they meet, independent of their observable productivity, and fire them in the next period if their true productivity is below a required level (to be determined later). However, in the presence of training costs, firms hire workers only if the observed z is such that the discounted value of the worker to the firm is at least equal to the hiring cost.

3.2 The financial contract

The financial market setup builds on costly-state-verification (Carlstrom and Fuerst, 1998). Firms use external finance for wage payments due to a cash flow mismatch as revenue is realized after wages are paid.[15] To obtain external finance, firms and lenders sign a financial contract based on the revenue of the firm subject to a shock ω_{jt}. Firm revenue depends on sales minus hiring and training costs and is given by $\omega_{jt}\big[\bar{y}_{jt}n_{jt} - C_v(v_{jt}) - c_\tau w^m p(\theta_t)(1 - H^z(z^*_{jt}))v_{jt}\big]$. Here, n_{jt}, and $\bar{y}_{jt}$ refer to firm j's number of employees and average output per employee, respectively. $p(\theta_t)(1 - H^z(z^*_{jt}))v_{jt}$ is the number of workers for which the firm pays training costs $c_\tau w^m$. The shock to firm revenue ω_{jt} cannot be observed by the lender without paying a monitoring cost $0 < \mu_{jt} < 1$. This monitoring cost μ_{jt} determines the extent of the financial constraints that a firm faces and is exogenous.[16] The shock ω_{jt} is *iid* across firms and time, has positive support $E(\omega_{jt}) = 1$ and the cumulative distribution $\mathcal{F}(.)$. The financial contract is signed before ω_{jt} is realized and the firm and the lender agree on a cutoff value $\bar{\omega}_{jt}$ such that if $\omega_{jt} > \bar{\omega}_{jt}$, the firm pays back $\bar{\omega}_{jt}\big[\bar{y}_{jt}n_{jt} - C_v(v_{jt}) - c_\tau w^m p(\theta)(1 - H^z(z^*_{jt}))v_{jt}\big]$ and keeps $(\omega_{jt} - \bar{\omega}_{jt})\big[\bar{y}_{jt}n_{jt} - C_v(v_{jt}) - c_\tau w^m p(\theta_t)(1 - H^z(z^*_{jt}))v_{jt}\big]$. If $\omega_{jt} < \bar{\omega}_{jt}$, the firm defaults and all revenue is claimed by the lender. The firm keeps its workers and continues to produce in the next period.

The expected gross share of revenue going to the lender is $\Gamma(\bar{\omega}_{jt}) = \int_0^{\bar{\omega}_{jt}} \omega d\mathcal{F}(\omega) + \int_{\bar{\omega}_{jt}}^{\infty} \bar{\omega}_{jt} d\mathcal{F}(\omega)$. Firms prefer to set the cutoff $\bar{\omega}_{jt}$ as low as possible, lenders favor a high cutoff.[17] Lenders only give credit if their expected return is at least the amount borrowed. Therefore, the lenders' participation constraint is

$$\big(\Gamma(\bar{\omega}_{jt}) - \mu_{jt}G(\bar{\omega}_{jt})\big)\Big[\bar{y}_{jt}n_{jt} - C_v(v_{jt}) - p(\theta)\big(1 - H^z(z^*_{jt})\big)c_\tau w^m\Big] = \bar{w}_{jt}n_{jt}. \tag{6}$$

Here, $\mu_{jt}G(\bar{\omega}_{jt}) = \mu_{jt}\int_0^{\bar{\omega}_{jt}} \omega d\mathcal{F}(\omega)$ describes the expected monitoring cost and $\bar{w}_{jt}$ are average wage payments. The optimal cutoff $\bar{\omega}_{jt}$ is determined in the firm maximization problem below.

[15] Here, firms cannot save and accumulate assets to avoid external finance. This is a common assumption in the literature to give a meaningful role to external finance and financial constraints. In Arabzadeh et al. (2020), we discuss cases where part of firm profits can be used to accumulate savings.

[16] In line with the literature, the idea is that firms cannot escape this financial constraint as they cannot switch banks easily due to information frictions and long-term customer relationships (see e.g., Sharpe, 1990).

[17] $\Gamma(\bar{\omega}_{jt})$ is increasing in the threshold $\bar{\omega}_{jt}$ and $\Gamma'(.) > 0$ and $\Gamma''(.) < 0$.

3.3 The firm problem

Firm j maximizes expected discounted profits taking as given aggregate labor market tightness θ, the monitoring cost μ and the minimum wage w^m

$$\max_{v_{jt},\bar{\omega}_{jt},n_{jt+1}} J(n_{jt}) = \left(1-\Gamma(\bar{\omega}_{jt})\right)\left[\bar{y}_{jt}n_{jt} - C_v(v_{jt}) - p(\theta)(1-H^z(z^*_{jt}))c_\tau w^m v_{jt}\right] + \beta E_t J_{jt+1}(n_{jt+1})$$

subject to the law of motion for employment (4) and the lenders' participation constraint (6).

The first-order condition with respect to the financial cutoff $\bar{\omega}_{jt}$ is given by

$$\phi_{jt} = \frac{\Gamma'(\bar{\omega}_{jt})}{\Gamma'(\bar{\omega}_{jt}) - \mu_j G'(\bar{\omega}_{jt})}, \tag{7}$$

where ϕ_{jt} is the Lagrangian multiplier on (6) and reflects the marginal cost of borrowing with $\frac{\partial \phi_j}{\partial \mu_j} > 0$. The external finance premium is given by the expected monitoring costs relative to the amount borrowed

$$\frac{\mu G(\bar{\omega}_{jt})\left[\bar{y}_{jt}n_{jt} - C_v(v_{jt}) - p(\theta)(1-H^z(z^*_{jt}))c_\tau w^m v_{jt}\right]}{\bar{w}_{jt}n_{jt}}. \tag{8}$$

The first-order condition with respect to employment defines reservation output y^*_{jt}

$$J'_{jt} = \Omega_{jt}y^*_{jt} - \phi_{jt}\bar{w}_{jt} + \beta(1-\delta)E_t J'_{jt+1} = 0, \tag{9}$$

where $J'_{jt} = \frac{\partial J_{jt}}{\partial n_{jt}}$ and

$$\Omega_{jt} = (1-\Gamma(\bar{\omega}_{jt})) + \phi_{jt}\left[\Gamma(\bar{\omega}_{jt}) - \mu_j G(\bar{\omega}_{jt})\right], \tag{10}$$

where Ω_{jt} measures how an increase in revenue affects the firm's value function. An increase in output relaxes the participation constraint of the financial contract (second term) and generates higher profits (first term).[18] The firing cutoff is determined by setting the marginal value of a worker J' to zero (i.e., firms fire if the marginal value turns negative). The first-order condition with respect to vacancies results in a free-entry condition

$$\beta p(\theta_t)\left(1-H^x_{jt}(x^*_{jt})\right)E_t J'_{jt+1} = \Omega_{jt}C'_v(v_{jt}) + \Omega_{jt}p(\theta_t)(1-H^z(z^*_{jt}))c_\tau w^m \tag{11}$$

[18] Note that $\frac{\partial \phi_j}{\partial \mu_j} > 0$ and $\frac{\phi_j}{\Omega_j} > 1$ and $\frac{\partial(\frac{\phi_j}{\Omega_j})}{\partial \mu_j} > 0$.

that equates the marginal benefit of a worker to the expected costs for vacancy creation and training.

The firm determines the hiring cutoff by equating the marginal value of hiring to zero (i.e., they hire if the marginal value is positive). This defines the hiring cutoff z^*_{jt} by

$$-\Omega_{jt} c_\tau w^m + \frac{\beta}{1-\beta(1-\delta)} \Big(\Omega_{jt} E[y_{ijt}|z^*_{jt}] - \phi_{jt} E[w_{ijt}|z^*_{jt}] \Big) = 0. \tag{12}$$

The first term on the right-hand side represents the effective training cost. $E[y_{ijt}|z_{ijt}]$ and $E[w_{ijt}|z_{ijt}]$ are the expected output and wage of the worker with observable productivity z_{ijt}. Hence, the second term represents the expected (discounted) value of the worker to the firm.[19]

3.4 Wage determination

Market wages are the outcome of individual Nash bargaining between firms and workers and as a result, are match-specific. Given individual bargaining and one employee's wage being an insignificant share of the total wage bill, neither the firm nor the worker considers the effect of wages on the firm's financial constraints when they negotiate the wage. The value function of an employed worker with output $y_{ijt} > y^*_{jt}$ is

$$W_{ijt} = w_{ijt} + \beta \big[(1-\delta) E_t W_{ijt+1} + \delta E_t U_{t+1} \big]. \tag{13}$$

Workers are paid the same wage for the same output independent of what share of output is determined by effort versus productivity. Hence, firms do not compensate workers for effort and the effort cost does not appear in the workers' value function W. The value function of unemployed individuals is given by

$$U_t = b + \beta \Big[f(\theta_t)\big(1 - H^x_{jt}(x^*_{jt})\big) E_t \bar{W}_{t+1} + \Big(1 - f(\theta_t)\big(1 - H^x(x^*_{jt})\big)\Big) E_t U_{t+1} \Big], \tag{14}$$

where $E_t \bar{W}_{t+1}$ is the expected average value of an employed worker in $t+1$.

Nash bargaining implies that the market wage is the solution to the following maximization

[19] The expectations are defined as $E[y_j|z^*_j] = \frac{1}{1-H^\epsilon(x^*_j - z^*_j)} \Big[\int_{\epsilon = x^*_j - z^*_j}^{x^m_j - z^*_j} (z^*_j + \epsilon) dH^\epsilon(\epsilon) + \int_{\epsilon = x^m_j - z^*_j}^{\infty} (z^*_j + \epsilon + e^*_j) dH^\epsilon(\epsilon) \Big] = \frac{1}{1-H^\epsilon(x^*_j - z^*_j)} \Big[\int_{\epsilon = x^*_j - z^*_j}^{\infty} (z^*_j + \epsilon) dH^\epsilon(\epsilon) + [1 - H^\epsilon(x^m_j - z^*_j)] e^*_j \Big]$ and $E[w_j|z^*_j] = \frac{1}{1-H^\epsilon(x^*_j - z^*_j)} \Big[\big(H^\epsilon(x^m_j - z^*_j) - H^\epsilon(x^*_j - z^*_j) \big) w^m + \int_{\epsilon = x^m_j - z^*_j}^{\infty} \Big[\tilde{w}(z^*_j + \epsilon + e^*_j) \Big] dH^\epsilon(\epsilon) \Big]$, where $\tilde{w}(.)$ is the market wage, see Equation (16).

problem where η is the workers' bargaining power

$$\max_{w_{ijt}} (J'_{jt})^{(1-\eta)}(W_{ijt} - U_t)^{\eta}.$$

This maximization problem implies that[20]

$$W_{ijt} - U_t = \frac{\eta}{\phi_{jt}(1-\eta)} J'_{jt}. \tag{15}$$

3.5 Labor market equilibrium in steady state and effort choice

The labor market equilibrium in steady state is given by the wage equation, the job creation condition and the hiring and firing cutoffs. The steady-state match-specific wage is given by[21]

$$\tilde{w}_j(y_{ij}) = w_j = \eta \frac{\Omega_j}{\phi_j} y_{ij} + (1-\eta)(1-\beta)U. \tag{16}$$

Wages depend on match-specific output-per-worker y_{ij} and are affected by the financial constraints that the firm faces - higher financial constraints imply a larger 'financial labor wedge' $\frac{\Omega_j}{\phi_j} < 1$ between productivity and wages[22] - and the outside option of workers in the bargaining U. The job creation condition is the steady state version of Equation (11)

$$\frac{\beta}{1-\beta(1-\delta)} p(\theta)\big(1 - H_j^x(x_j^*)\big)(\bar{y}_j - \frac{\phi_j}{\Omega_j}\bar{w}_j) = C'_v(v_{jt}) + p(\theta)\big(1 - H^z(z_j^*)\big)c_\tau w^m. \tag{17}$$

Steady-state employment is given by

$$\delta n_j = p(\theta) v_j \big(1 - H_j^x(x_j^*)\big) = f(\theta)(1-n_j)\big(1 - H^z(z_j^*)\big) - f(\theta)(1-n_j)\big(H_j^x(x_j^*) - H^z(z_j^*)\big). \tag{18}$$

Employment is constant if the exogenous separations δn_j equal hirings $f(\theta)(1-n_j)\big(1-H^z(z_j^*)\big)$ net of endogenous firings $f(\theta)(1-n_j)\big(H_j^x(x_j^*) - H^z(z_j^*)\big)$.

Hiring and firing in the labor market are determined by the various productivity cutoffs as

[20] The wage derivation is presented in detail in Appendix B.

[21] See Appendix B for the derivation.

[22] In Arabzadeh et al. (2020), we discuss that this wedge appears also in more general model versions.

		Effort e_i	Output y_i	Wage w_i
(1)	$x_i < x^* =$ *firing threshold*	0	-	-
(2)	$x^* \leq x_i < y^* =$ *reservation output*	$y^* - x_i > 0$	y^*	w^m
(3)	$y^* \leq x_i < x^m = w^m$ *threshold*	0	x_i	w^m
(4)	$x^m \leq x_i$	$e^* > 0$	$x_i + e^* \geq y^m$	$w_i > w^m$

Table 3: Overview of productivity and output thresholds in the model for a given firm j.

determined above. From Equation (9) follows the reservation output y_j^* in steady state as

$$y_j^* = \frac{\phi_j}{\Omega_j} w^m. \tag{19}$$

The steady state version of Equation (12) determines the hiring cutoff z_j^*. The steady state of the firing cutoff x^* depends on the effort that workers exert. Workers choose their effort taking into account which wage corresponds to the respective output that they produce (given by Equation (16)), but they do not account for the share of productivity vs. effort (see Equation (13)).

Optimal effort can be described in terms of four cases that depend on match productivity x_{ij}. Table 3 provides an overview. In firm j, a low-productivity worker i with $x_{ij} < y_j^*$ and no effort would produce output below the reservation output and, hence, will be fired (case 1). If she exerts effort, however, she can remain on the job if her effort is at least equal to $y_j^* - x_{ij}$ (case 2). She would exert this effort only if the monetary benefit of this effort outweighs its cost. This determines the firing productivity threshold $x_j^* < y_j^*$ below which workers find it optimal to exert no effort and be fired, while workers with productivity $x_j^* < x_{ij} < y_j^*$ exert $e_{ij} = y_j^* - x_{ij}$, and remain on the job. These workers provide zero profit and receive the minimum wage w^m. The threshold x_j^* is the solution to the following equation

$$w^m - C(y_j^* - x_j^*) = (1 - \beta)U. \tag{20}$$

The threshold is defined such that the benefit from working equals the reservation wage (value of unemployment). Without effort, $x_j^* = y_j^*$.

We next define y^m as the output-per-worker for which the associated market wage is exactly equal to the minimum wage. y^m is the solution to the following equation

$$w^m = \tilde{w}_j(y^{ij}) = \eta \frac{\Omega_j}{\phi_j} y_j^m + (1 - \eta)(1 - \beta)U. \tag{21}$$

For output above y_j^* and up to y_j^m, workers have no incentives to exert effort, since they always receive the minimum wage w^m (case 3). A firm with financial constraints will hence have a higher y^m compared to a firm without financial constraints, i.e., conditional on productivity it pays more workers the minimum wage.

Above y^m higher output leads to higher wages and, hence, effort pays off (case 4). Using Equations (16) and (21) one obtains

$$C(e_j^*) = \eta \frac{\Omega_j}{\phi_j}(x_j^m + e_j^* - y_j^m). \tag{22}$$

The optimal level of effort, e^*, is the level at which the marginal cost of effort equals the marginal benefit

$$C_e'(e_j^*) = \eta \frac{\Omega_j}{\phi_j}. \tag{23}$$

Optimal effort e_j^* does not depend on match-productivity x_{ij} (as wages increase linearly in output, see Equation (16)), but falls with higher financial constraints in the firm (given that constrained firms reward effort by less). The productivity cutoff above which workers start exerting effort, x_j^m, is determined by

$$w^m = \tilde{w}(x_j^m + e_j^*) - C(e_j^*). \tag{24}$$

3.6 Comparative statics

In this section, we derive comparative statics to a change in the minimum wage, a change in financial conditions and the interaction of the two for vacancy creation, hiring, the different productivity cutoffs and wage dispersion. Derivations and proofs can be found in Appendix C.

Without effort, an increase in the minimum wage, ceteris paribus, increases the cost of hiring and therefore decreases the incentives to post vacancies. Generally, the higher the minimum wage, the lower is the expected value of the worker to the firm. This increases the hiring cutoff z^* and the firm hires less (see Equation (12)). The minimum wage then unambiguously increases the reservation output y^* (see Equation (19)). Without effort, the reservation output y^* defines the firing cutoff x^*. Hence, fewer low-productivity workers remain employed when the minimum wage increases. Together with lower hiring, an increase in the minimum wage then unambiguously decreases employment. We refer to this as the *selection channel* of the minimum wage.

With effort, a higher minimum wage encourages more low-productivity workers to exert effort and remain employed (case 2 in Table 3). Then, the firing cutoff x^* increases less or may even decline such that an increase in the minimum wage can increase employment. We refer to this as the *effort channel* of the minimum wage. One factor driving effort is the value of unemployment which increases in the expected wage and labor market tightness. The value of unemployment may hence rise or fall with the minimum wage depending on the wage vs. employment effects.[23] Note that the optimal effort of non-minimum wage employees does not depend on the minimum wage (see e^* in Equation (23)). However, the minimum wage increases the minimum wage threshold x^m.[24] Hence, some medium-productivity workers are now paid the minimum wage that received a market wage above the minimum wage before (more workers in case 3 in Table 3). These workers then exert lower effort.

An increase in monitoring costs μ worsens the financial conditions for firms. A part of these additional costs is shifted to workers in the Nash bargaining and, hence, wages are lower when financial constraints increase ('financial labor wedge', see Equation (16)). A binding minimum wage hinders this adjustment. This implies that, with a minimum wage, the costs of borrowing ϕ increase by more for a given increase in monitoring cost μ. Financial constraints, therefore, reduce the expected profit from a worker more, reduce the incentive for firms to post vacancies (see Equation (11)) and to hire (z^* increases, see Equation (12)). Higher financial constraints increase the effective cost of the minimum wage to firms, and, hence, increase their reservation output y^* and the firing cutoff x^*.

Minimum wages and financial constraints therefore intensify each other. Since financial constraints increase reservation output and the firing cutoff, they reinforce the selection channel of the minimum wage. Since wages are generally lower, financial constraints reduce the incentives to exert effort, i.e., they increase the threshold x^m, and decrease optimal effort e^* (compare Equations (22) and (23)). As a result, financial constraints buffer the effort channel.

Wage dispersion is affected by (1) the realized productivity distribution of employed workers (in particular the productivity threshold x^*), and (2) the aggregate labor market conditions. A

[23]Formally, the firing cutoff x^* may fall when the value of unemployment decreases after an increase in the minimum wage (or increases only a little, see Equation (20)).

[24]Formally, this holds if the value of unemployment does not increase too much after an increase in the minimum wage. In our quantitative exercises, this condition always holds. If the value of unemployment and as a result market wages rise a lot with the minimum wage, the minimum wage threshold x^m does not necessarily rise.

binding minimum wage affects both of these aspects. A more narrow productivity distribution (a higher x^*) implies lower wage dispersion since workers conditional on their productivity move down in the wage distribution. Conversely, a wider productivity distribution (a lower x^*) implies higher wage dispersion. More adverse aggregate labor market conditions (lower employment) worsen the outside option of workers in the bargaining for market wages. This reduces wages of non-minimum wage takers and wage dispersion decreases. Without effort, employment falls with the minimum wage and wage dispersion hence unambiguously decreases with the minimum wage. With effort, an increase in the minimum wage induces higher effort of low-productivity workers which increases productivity dispersion and might increase employment. Higher effort then increases wage dispersion through (1) and, potentially, through (2).

Our model therefore implies a negative trade-off between employment and wage dispersion. If the selection channel is strong, a higher minimum wage adversely affects employment, but reduces wage dispersion. If the effort channel is strong, a higher minimum wage might increase employment, but increases wage dispersion. Financial constraints enforce the selection channel and buffer the effort channel. Wage dispersion will hence always decrease more with higher minimum wages when financial constraints are present. This and the trade-off between employment and wage dispersion are both in line with the data.

4 Quantitative model analysis

In the following, we use a quantitative version of our model to quantify the selection and the effort channel of the minimum wage. We start with a baseline model where we impose a zero effect of the minimum wage on employment (in line with the empirical findings of Dustmann et al. (2022) for the German minimum wage introduction). However, given that the employment effect of minimum wages is empirically unclear, we discuss different model versions where we allow for a negative and positive employment effect, in turn.

We provide a steady state comparison, i.e., we analyze the long-run response to the minimum wage. First, we analyze an economy without financial frictions ($\mu = 0$) and with no minimum wage, where we then introduce a binding minimum wage.[25] We set the minimum wage such that the share of minimum wage takers equals 15% (see Dustmann et al., 2022). Second, we compare how a firm that faces a positive monitoring cost ($\mu > 0$) reacts differently to a similar

[25] In the model without a minimum wage $y^* = x^*$, y^m and x^m are irrelevant and the training cost is defined as $c_\tau w^{lowest}$, where w^{lowest} is the lowest market wage.

rise in the minimum wage as compared to the other firms that face zero financial constraints. In other words, we compare firms that operate in the same market, but face different financial constraints. Here, we assume that firms with financial constraints are few, i.e., we keep the aggregate variables (e.g., labor market tightness (θ) and the value of unemployment (U)) fixed. Third, we discuss the implications of a minimum wage if all firms, i.e., the whole market, are affected by financial constraints. Then, financial frictions affect the pass-through of the minimum wage to the aggregate economy.

4.1 Calibration

We start with targeting the German economy before the introduction of the minimum wage. Table 4 summarizes our calibration and targets. Given our interest in wage dispersion, our model should replicate wage dispersion as in our data. We use the ratio of the in-firm mean wage of full-time workers to the 10th percentile in 2011-2014 which is equal to 0.3 as a target. As outlined above, the wage distribution in the model is related to the distribution of match productivity x_{ij}, aggregate labor market conditions and the value of unemployment (that depends on unemployment benefits b). Match productivity is the sum of the observable productivity z_{ij} and the observational error ϵ_{ij}. We set a mean of zero for both distributions. We normalize the variance of the observational error ϵ to unity and then set the variance of observable productivity, σ_z^2, together with b to target the wage dispersion. This results in benefits of just 18% of the lowest market wage in the model without the minimum wage, a low level compared to an actual replacement rate of above 60% in Germany. A higher level is inconsistent with our target on wage dispersion as this would require a very wide productivity dispersion.[26] We address this issue in a robustness check in Section 4.4. Our next target is an average quarterly separation rate of about 1.5%. (Kuhn et al., 2018). We set the exogenous separation rate, δ, such that the total separation rate (endogenous firing plus exogenous separation relative to employment) matches this number.

The quarterly discount factor, β, is set to 0.993 in line with an average annual interest rate of 2.7% in 2006-2014 (source: OECD). The average unemployment rate in Germany between 2011 and 2014 is 7.2% (Kuhn et al., 2018). We use the matching efficiency parameter, ξ, to match this number. Following the literature, we set the matching function parameter ε to 0.72 and the

[26]In our model, productivity does not translate one-to-one into wages according to the Nash bargaining and bargaining power $0 \leq \eta \leq 1$. As high benefits reduce the incentives to work at a low wage, the implied wage dispersion is much smaller in a model with high benefits than in the data.

Parameter	Value	Explanation	Target
μ_z	0	mean of z_i	normalization
σ_z	0.65	std.dev. of z_i	wage dispersion $(log(\bar{w}) - log(w_{10}) = 0.3)$
σ_ϵ	1	std.dev. of ϵ_i	normalization
δ	0.0022	exogenous separation rate	separation rate 1.5% (Kuhn et al., 2018)
b	0.175	unemployment benefit	wage dispersion $(log(\bar{w}) - log(w_{10}) = 0.3)$
η	0.5	workers' bargaining power	Balleer et al. (2016)
ξ	0.1345	efficiency of matching function	unemployment rate 7.2% (Kuhn et al., 2018)
ε	0.72	matching elasticity	Balleer et al. (2016)
γ	10.639	vacancy cost	$\theta = 1$
β	0.99	discount rate	annual interest rate 2.76%
c_e	9.035	parameter of effort cost	zero employment effect of w^m
c_τ	0.3	parameter of training cost	30% of x^* (Mortensen and Pissarides, 1999)
μ_ω	1	mean of shock to firm revenue	normalization
σ_ω	0.086	std.dev. of shock to firm revenue	default rate 0.45%
μ	0.546	monitoring cost	external finance premium 0.24%

Table 4: Calibration of parameters (quarterly).

workers' bargaining power to 0.5 (see, for example, Balleer et al., 2016). Vacancy costs follow a quadratic function $C_v(v) = 0.5\gamma v^2$. Normalizing $\theta = 1$, γ then follows from the job-creation condition. We follow Mortensen and Pissarides (1999) and set training costs to be 30% of the lowest worker's productivity (i.e., $c_\tau w^{lowest} = 0.3x^*$, where w^{lowest} and x^* are, respectively, the lowest market wage and lowest productivity in the model without the minimum wage). We define the effort cost with a quadratic function: $C_e(e) = c_e e^2$.

For the financial constraints, we target a quarterly default rate of 0.45% and an external finance premium of 0.24%. These numbers follow Arabzadeh et al. (2020) for Germany.[27] This implies setting $\mu = 0.546 > 0$ and assuming a Normal distribution for the shock ω, a mean of zero and $\sigma_\omega = 0.086$.

4.2 Baseline model

In this model, we set the effort cost such that the employment effect of the minimum wage is zero in non-financially constrained firms. This implies $c_e = 9.04$. We discuss the introduction of a minimum wage without and with financial constraints in turn, Table 5 summarizes all results.

No financial constraints. Column 1 vs. column 3 of Table 5 show the effects of introducing a minimum wage for firms without financial constraints. According to the selection channel, firms create fewer vacancies and reduce hiring (z^* rises). Figure 2a illustrates how effort and output

[27] For comparison, Bernanke et al. (1999) target an annual default rate of three percent and a premium of two percent for the US.

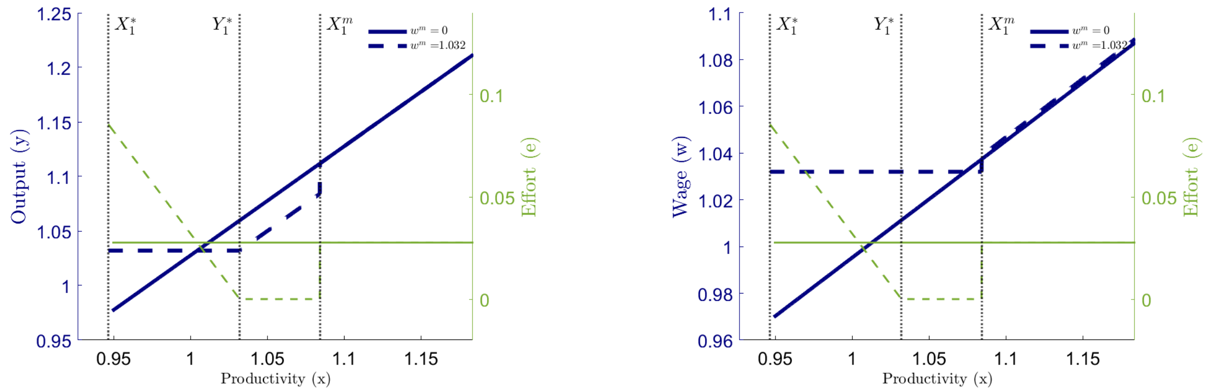
	No minimum wage ($w^m = 0$)		Binding min. wage ($w^m = 1.03 > 0$)	
Variable	Base	Financial frictions	Base	Financial frictions
Employment	0.928	0.881	0.928	0.875
Vacancies posted	0.072	0.070	0.071	0.069
Hiring ($\times 10$)	0.093	0.090	0.092	0.089
Firing ($\times 10$)	0.072	0.070	0.071	0.069
Av. wage	1.309	1.300	1.314	1.305
Av. productivity	1.628	1.642	1.626	1.643
Av. output (per worker)	1.655	1.670	1.653	1.670
w^m takers (share)	-	-	0.15	0.153
Finance premium	-	0.002	-	0.003
x^*	0.949	0.968	0.947	0.968
y^*	0.977	0.995	1.032	1.054
z^*	-1.127	-1.108	-1.085	-1.064
e^* ($\times 10$)	0.2767	0.2714	0.2767	0.2710

Table 5: Steady state comparison across different models.

change with the minimum wage conditional on match productivity. Workers with medium productivity ($y^* \leq x_i < x^m$) become minimum wage takers and reduce their effort and output. Low-productivity workers with $x^* \leq x_i < y^*$ increase effort and output to remain on the job. This implies that x^* falls slightly such that overall employment stays constant even though there are fewer hirings. As a result, the effort effect buffers the selection effect, in this case, the two exactly cancel.

Figure 2b displays the effect of the minimum wage on wages conditional on match productivity. Without the minimum wage (solid line), market wages increase linearly in match output and productivity (effort is constant in productivity). The minimum wage introduces a wage floor. As discussed above, low-productivity workers now exert higher effort to become eligible for the minimum wage, whereas medium-productivity workers reduce their effort. Just below the minimum wage threshold, there are some workers who prefer to become minimum wage takers and to not exert effort anymore. These workers receive a lower wage after the introduction of the minimum wage. Further, there are spillovers to wages for non-minimum wage takers. These increase slightly as the outside option in the wage bargain improves (due to a higher expected wage upon hiring).

To assess the effect on wage dispersion, one has to consider the wage conditional on productivity jointly with the worker reallocation. Here, given that we focus on a case with an employment effect of zero, the change in worker productivity of the employed is small (x^* hardly moves). This will change in the counterfactuals in the next subsection. Here, instead,



(a) **(b)**

Figure 2: Output (panel a), wages (panel b) and effort conditional on productivity in the counterfactual economies without ($w^m = 0$, solid) and with a minimum wage ($w^m > 0$, dashed); no financial constraints ($\mu = 0$).

one can interpret the effect on wage dispersion conditional on almost no reallocation in line with Figure 2b. Table 6 summarizes changes in the wage deciles with the minimum wage introduction. The quantitatively largest effect on the wage distribution comes from the increase in wages in the lowest percentile. Given that the effect on higher percentiles is smaller in comparison, this reduces wage dispersion, in this case by 1.6 percentage points. Figure 3a illustrates the wage distribution and shows that this distribution is basically identical to the case without a minimum wage, the minimum wage only implies a bunching at the wage floor.

Introducing financial constraints. Next, we introduce financial constraints for one firm in this economy. In line with our previous arguments, Table 5, column 2 shows that a firm with tighter financial constraints has lower employment, fewer vacancies, and fewer hires compared to a non-constrained firm (column 1). Figure 3b shows that the lower bound of the realized productivity distribution, i.e., the productivity threshold x^*, shifts to the right (red vs. blue line). Due to higher selectivity, average productivity and output-per-worker are higher in this firm compared to non-constrained firms, whereas effort and wages are lower (financial labor wedge). This holds across all wage deciles (see Table 6). Given that constrained firms reward higher output by disproportionately less, they have lower wage dispersion.

Minimum wages and financial constraints. Comparing columns 2 and 4 of Table 5 shows the effects of a minimum wage for a firm with financial constraints. Given that the constrained firm pays ex-ante lower wages, the firm experiences a higher wage increase due to the minimum wage by construction (the minimum wage is more binding). This is illustrated by the red bars in Figure 3d that show the wage change due to the introduction of the minimum wage across

Variable	No minimum wage ($w^m = 0$) Base	Financial frictions	Binding min. wage ($w^m = 1.03 > 0$) Base	Financial frictions
Wage deciles	*Level*		$\Delta\%$	
w_{10}	1.02	1.01	1.67	1.83
w_{25}	1.09	1.09	0.04	0.06
w_{50}	1.24	1.23	0.05	0.04
w_{75}	1.46	1.45	0.06	0.00
w_{90}	1.70	1.69	0.06	-0.02
Wage dispersion	*Level*		Δ (pp's)	
$log(w_{90}) - log(w_{10})$	0.52	0.51	-1.61	-1.83

Table 6: Steady state comparison of wage deciles and wage dispersion in baseline economies. Changes are computed relative to the case with no minimum wage.

the wage deciles. The workers in the lower deciles benefit the most from the minimum wage. In contrast, the top deciles are relatively worse off (i.e., they benefit less). The reason is that the minimum wage increases the wage bill and because of that firms' financial constraints. This deepens the financial labor wedge and reduces wages for workers with market wages.

Financial constraints intensify the selection channel (firms post fewer vacancies and hire less on net, see Table 5) and dampen the effort channel (financial constraints increase and reduce wages). For the given effort parameterization, this implies that the selection effect now dominates the effort effect and firing falls not enough to compensate for the reduction in hiring. As a consequence, with financial constraints, employment falls with the minimum wage. Likewise, average output-per-worker falls by less such that firms with financial constraints trade off lower employment versus relatively higher average output-per-worker.

All these effects contribute to the model's implication that wage dispersion falls by more with the minimum wage in financially constrained firms. The stronger selection channel shrinks the realized (conditional) productivity, and hence, wage distribution. A higher minimum wage increases the financial constraints in firms and as a result, weakens the effort channel and raises the financial labor wedge for non-minimum wage takers. According to Table 6, wage dispersion falls by 1.8 percentage points in firms with financial constraints, whereas it falls by 0.2 percentage points less, i.e., by 1.6 percentage points, without financial constraints. In the data (compare Section 2.3), we estimate a similar drop in in-firm wage dispersion of 1.8 percentage points due to the minimum wage introduction in firms with a minimum wage bite of 20% and without financial constraints. The additional effect with financial constraints is considerably larger and ranges between 0.3 and 1.6 percentage points for different levels of leverage.

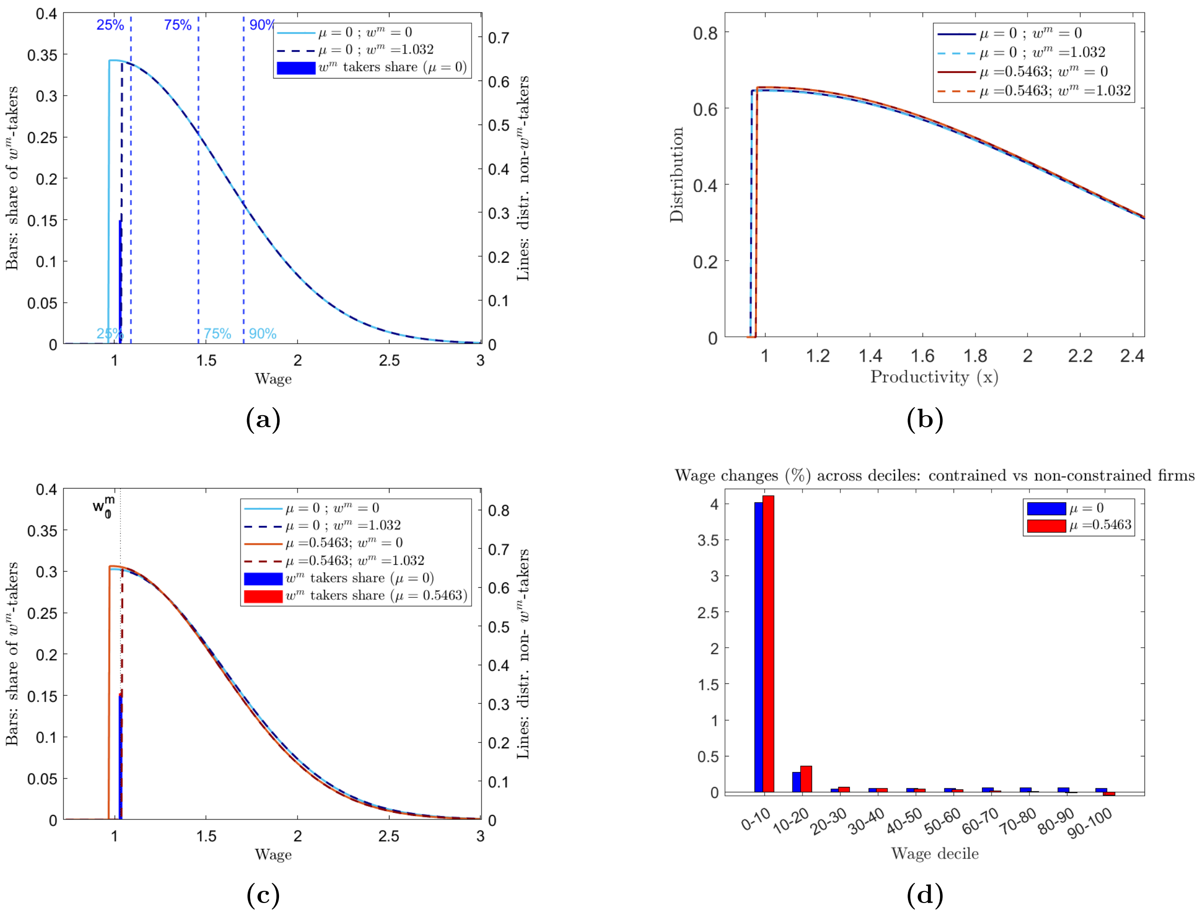


Figure 3: Wage and productivity distributions and wage deciles in **baseline** model. Blue without financial constraints ($\mu = 0$), red with financial constraints ($\mu > 0$). Panels a and c: solid lines without minimum wage, dashed lines with minimum wage. The wage changes in panel d show the change in the average wage of workers in each percentile bin.

4.3 Counterfactual exercises

The purpose of this section is to discuss counterfactual situations where we modify the aggregate employment effect of the minimum wage and to analyze the role of the effort channel. Table 7 shows the effects on wage dispersion, Figure 4 illustrates the most important results, Appendix Table A.6 summarizes all results.

4.3.1 Negative employment effect

In this model, we set the effort cost to $c_e = 20$ such that effort is very costly. This implies that the effort channel is basically absent from the model such that the aggregate employment effect of the minimum wage turns negative with -0.3 percentage points (the effort effect does

Variable	No minimum wage ($w^m = 0$) Base	No minimum wage ($w^m = 0$) Financial frictions	Binding min. wage ($w^m = 1.03 > 0$) Base	Binding min. wage ($w^m = 1.03 > 0$) Financial frictions
Baseline: Zero employment effect				
$log(w_{90}) - log(w_{10})$	0.519	0.509	0.503	0.491
Δ (pp's)			*-1.61*	*-1.83*
$log(w_{75}) - log(w_{25})$	0.292	0.286	0.292	0.286
Δ (pp's)			*0.01*	*-0.06*
Counterfactual 1: Negative employment effect				
$log(w_{90}) - log(w_{10})$	0.522	0.512	0.494	0.484
Δ (pp's)			*-2.78*	*-2.88*
$log(w_{75}) - log(w_{25})$	0.294	0.288	0.289	0.284
Δ (pp's)			*-0.46*	*-0.47*
Counterfactual 2: Positive employment effect				
$log(w_{90}) - log(w_{10})$	0.506	0.498	0.513	0.500
Δ (pp's)			*0.63*	*0.23*
$log(w_{75}) - log(w_{25})$	0.285	0.280	0.291	0.284
Δ (pp's)			*0.58*	*0.45*

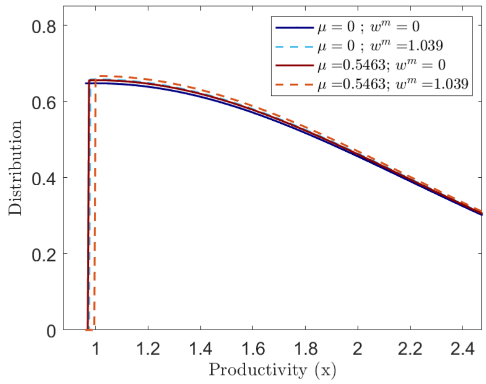
Table 7: Steady state comparison of wage dispersion in counterfactual economies.

not compensate for the selection effect anymore).[28]

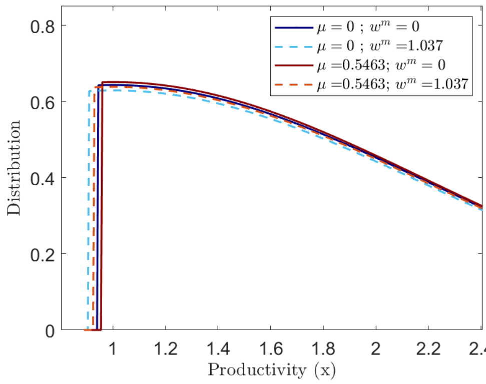
According to Table 7, wage dispersion keeps falling, by even more compared to the baseline. This is due to the selection effect that compresses the realized productivity distribution (see Figure 4a) and wage dispersion as low-productivity workers lose their jobs. The buffering from the effort channel is absent now. Interestingly, wage dispersion of non-minimum wage takers now also falls. We measure wage dispersion of non-minimum wage takers using the log difference of the 75th-to-25th wage percentile (with 15 percent minimum wage takers, the 25th percentile does not include minimum wage takers). In contrast, in our baseline model with a zero employment effect wage dispersion of non-minimum wage takers actually increased slightly (without financial constraints).

Irrespective of this finding, the effect of financial constraints on wage dispersion is the same as before: The reduction is stronger for financially constrained firms. Financial constraints intensify the selection effect and wage dispersion falls by more.

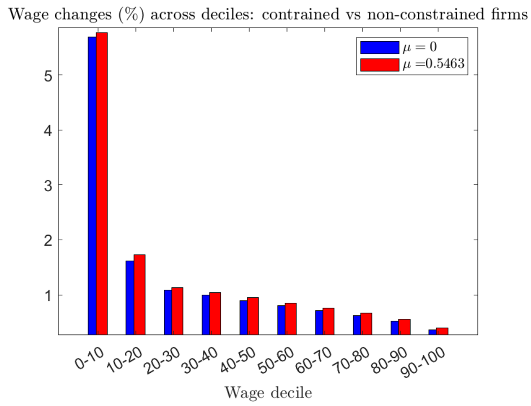
[28] Firing increases so much in this case that due to an equilibrium effect (unemployment rises, tightness falls), vacancy creation rises. See Appendix Table A.6.



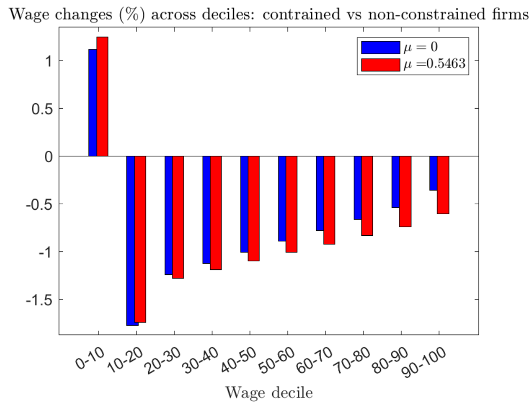
(a)



(b)



(c)



(d)

Figure 4: Productivity distributions and wage deciles in **counterfactual** models (a and c: for negative employment effect, b and d: for positive employment effect). The wage changes in panels c and d show changes in the average wage of workers in each percentile bin.

4.3.2 Positive employment effect

Next, we target a positive employment effect of the minimum wage by setting the effort cost to a relatively low level with $c_e = 3$. Then, the effort effect becomes so strong that it dominates the selection effect and employment rises by 0.3 percentage points with the minimum wage. This changes the effects of the minimum wage on wage dispersion. Wage dispersion now rises irrespective of whether we include minimum wage takers or not if there are no financial constraints (Table 7). The minimum wage triggers such an increase in effort that firms keep more low-productivity workers and aggregate employment rises. This shifts the productivity cutoff to the left (see Figure 4b) and the realized productivity distribution widens. This shift of the productivity distribution implies that the wage distribution shifts as well, see Figure 4d which shows that wages in almost all wage percentiles fall. Note that this does not imply that the wages of workers conditional on their productivity actually fall. Instead, we observe that workers move up in the distribution when more low-productivity workers are hired (reallocation). Because of this, the minimum wage implies a wage increase only in the lowest wage percentiles (see Figure 4d). These low percentiles are not covered in the common measures of wage dispersion that we use (we look at the 10th-percentile at minimum) such that this increase is not visible.[29]

When taking into account financial constraints, wage dispersion including the minimum wage takers (90-to-10th percentile) falls again in line with our earlier results. As before, selection is intensified and the effort effect is buffered.[30]

In sum, the model predicts that while the effect of the minimum wage on wage dispersion is ambiguous and depends on the importance of the effort channel, the effect of financial constraints is unambiguous: They imply a stronger reduction or less of an increase in wage dispersion. Since vacancy creation declines with the minimum wage, the employment effect of the minimum wage can only be positive if firms become less selective (x^* declines). This implies that as long as the employment effect is positive, wage dispersion among non-minimum-wage takers always increases with the minimum wage. As a result, our model highlights a trade-off between the effect of the minimum wage on employment and on wage dispersion.

4.4 Alternative calibration

Before, we targeted wage dispersion in line with the data. As discussed, this requires a relatively low unemployment benefit b. Here, instead, we discipline the productivity distribution using worker-level fixed effects following the framework proposed by Abowd et al. (1999) (AKM). These are the outcomes of a regression of log earnings on worker fixed effects, establishment fixed effects, time-varying covariates, and idiosyncratic error terms. Worker-fixed effects can be interpreted as a measure of individual skill or productivity and establishment-fixed effects represent the proportional pay premia at this level. For our purposes, we use the worker-level AKM effects for 2007-2013 as estimated by Lochner et al. (2023) to compute the within-firm productivity distribution. We find that among full-time employees within a firm, the ratio of the mean-to-10th percentile worker AKMs is 0.3.[31] We set the parameter σ_z^2 such that we match

[29] The worker at the 10th-wage percentile receives a wage above the minimum wage before the minimum wage introduction. With the minimum wage, this worker moves to a higher percentile (above the 15th-wage percentile) as more low-productivity workers stay on their jobs.

[30] This buffering of the effort channel is so strong, that employment falls for financially constrained firms in this case (see Appendix Table A.6).

[31] In the data, wage and productivity dispersion (as measured using the AKM fixed effects) are very closely related. This is not true in the model with Nash bargaining where productivity drives only part of the wage,

this statistic, while this allows to set the replacement rate to 60% (in line with the German unemployment insurance system).

Appendix Table A.6 summarizes the results. With this parameterization. As before, wage dispersion falls, and it falls more in financially constrained firms. Quantitatively, the differences are smaller compared to our baseline. Focusing on wage dispersion of workers not directly affected by the minimum wage ($log(w_{75}) - log(w_{25})$), we again find that wage dispersion of these workers increases (as in Section 4.3.2), but the effect is - echoing the earlier results - smaller in financially constrained firms.

4.5 Financial constraints on the aggregate level

So far, we have focused on a single firm that acts in a market that is not affected by financial constraints (the single firm does not affect aggregate labor market conditions). Next, we investigate the effect of a minimum wage if it is introduced in a market where all firms are affected by financial constraints (as e.g., in a financial crisis). Then, financial constraints affect the aggregate labor market which feeds back into wages and wage dispersion.

Calibrated to the same targets, employment and wage dispersion are the same in the models with and without financial constraints. The minimum wage now affects all firms as described in Section 4.2. As presented in Appendix Table A.6 (panel a versus panel c), firms post fewer vacancies and fire more compared to the baseline economy without financial constraints.[32] This implies lower aggregate employment which then feeds back into firm decisions. This is not the case when only one firm is financially constrained (panel b). Lower employment improves incentives for firms to create vacancies which buffers the adverse employment effect (in panel c compared to panel b). Note that the average wage in the economy with financial constraints is lower than in the baseline economy, since part of the financial cost is burdened by workers. A lower average wage before the minimum wage introduction intensifies the effect on wage dispersion as the lower end of the wage distribution increases by more. A lower average wage affects the value of unemployment negatively which enhances effort. This further buffers the fall in employment. If only a single firm is financially constrained in the market (panel b),

the remainder depends on the outside option of workers in the bargaining. With a high unemployment benefit, productivity is relatively less important for the wage and the wedge between productivity and wages becomes larger. This is why we can either match the productivity distribution and the unemployment benefit, or the wage dispersion.

[32] They increase their hirings, as conditional on finding a worker (which is less likely given fewer vacancies), they keep more workers.

average wages are higher and the reduction in wage dispersion is less pronounced. At the same time, effort is buffered more which results in a stronger fall in employment relative to the economy in which all firms are financially constrained. In sum, the selection effect is intensified (employment is reduced by less and minimum wage affectedness is higher, such that financial constraints increase by more). As a result, wage dispersion falls by more with financial constraints. This confirms our earlier findings while taking into account that the aggregate feedback effects reinforce this pattern. Quantitatively, we observe a reduction in wage dispersion by 3.1 percentage points with financial constraints which is close to our empirical estimate of $1.8 + 1.6 = 3.4$ percentage points for very high leverage.

5 Conclusions

This paper examines the interaction between minimum wages, wage dispersion and firms' financial constraints and sheds light on the heterogeneous effects of minimum wage increases across firms. Our empirical analysis builds on the introduction of a federal minimum wage in Germany in 2015 and reveals a clear reduction in within-firm wage dispersion, particularly for financially constrained firms. Building on these findings, we develop a theoretical labor market model, which incorporates both a selection effect and an effort effect of the minimum wage. Whereas selection reduces employment and wage dispersion in response to a minimum wage, the effort effect may increase employment and wage dispersion. Depending on effort versus selection, the model hence highlights a trade-off between the employment effect of a minimum wage and the effect on wage dispersion. The model demonstrates that financial constraints intensify the selection channel of the minimum wage, leading to lower wage dispersion and lower employment. The effort channel is weakened by financial constraints as firms transfer part of the wage costs to workers. This further reduces wage dispersion.

The results emphasize the importance of firms' financial conditions for understanding the distributional impacts of minimum wage policies. Additionally, our study suggests that the effects of minimum wage changes may vary depending on the economic environment, such as financial recessions or booms. Overall, this study contributes to the ongoing debate on how minimum wages affect the income distribution by providing insights into the complex relationship between minimum wages and firm-level outcomes.

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A Additional empirical results

	Control (minwage bite<=0.05)		Partial treatment (0.05<minwage bite<=0.15)		Treatment (minwage bite>0.15)	
Time period	low lever.	high lever.	low lever.	high lever.	low lever.	high lever.
Productivity (worker-AKMs), in logs						
2011-2014	4.5542	4.4700	4.3373	4.3319	4.2817	4.2718
2015-2017	4.5405	4.4588	4.3346	4.3313	4.2847	4.2762
Difference	-0.0137	-0.0112	-0.0027	-0.0006	0.0030	0.0043
Employment (full-time), in logs						
Difference	0.0221	0.0017	0.0090	-0.0146	-0.0178	-0.0714

Table A.1: Summary statistics on firm-level median worker-AKMs and employment pre- and post-minimum wage. Difference compares the post-policy to the pre-policy variables. High-leverage firms are those whose average debt-to-assets ratio before 2015 is above 50 percent.

Wage percentiles		log wage p10			log wage p25			log wage p50	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
minwage bite × post-2014	0.2786***	0.1763***	0.1216***	0.2205***	0.1319***	0.0798***	0.1701***	0.0964***	0.0456**
	0.017	0.0087	0.0352	0.0146	0.0065	0.0255	0.014	0.0056	0.022
leverage × post 2014	0.0036	0.0004	-0.0070***	-0.0014	-0.0030***	-0.0045***	-0.0045	-0.0040***	-0.0040***
	0.0032	0.0015	0.0018	0.0033	0.0011	0.0012	0.0037	0.0011	0.0011
minwage bite × leverage × post-2014	0.0609**	0.0974***	0.0908***	0.0757***	0.0935***	0.0661***	0.0644**	0.0772***	0.0521***
	0.031	0.0185	0.0191	0.0269	0.0138	0.0128	0.0263	0.0115	0.011
Wage percentiles		log wage p75			log wage p90				
	(1)	(2)	(3)	(4)	(5)	(6)			
minwage bite × post-2014	0.1340***	0.0721***	0.0566***	0.1379***	0.0878***	0.0681***			
	0.0145	0.0052	0.0205	0.016	0.0061	0.0247			
leverage × post 2014	-0.0026	-0.0023**	-0.0026**	0.0062	0.0046***	0.0009			
	0.0039	0.0011	0.0012	0.0042	0.0013	0.0014			
minwage bite × leverage × post-2014	0.0465*	0.0602***	0.0324***	0.0096	0.0183	-0.0037			
	0.0274	0.0105	0.0109	0.0301	0.0129	0.0149			
Fixed Effects	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes
Firm Controls	No	No	Yes	No	No	Yes	No	No	Yes
Observations	1,501,901	1,487,404	1,091,446	1,501,901	1,487,404	1,091,446	1,501,901	1,487,404	1,091,446

Table A.2: Effect of financial constraints on wage percentiles.

Wage dispersion	log wage p75-p25		log wage p90-p10	
	(1)	(2)	(3)	(4)
minwage bite × 2012	0.0145**	-0.0006	0.0119	0.0582
	0.0066	0.0255	0.0097	0.0447
minwage bite × 2013	0.0161***	-0.0064	0.0220***	0.0417
	0.0058	0.0209	0.0076	0.0362
minwage bite × 2015	-0.0485***	0.0151	-0.0799***	0.0059
	0.0052	0.0233	0.0089	0.0409
minwage bite × 2016	-0.0487***	-0.0293	-0.0760***	-0.0348
	0.0068	0.0265	0.0102	0.0455
minwage bite × 2017	-0.0535***	-0.0526*	-0.0802***	-0.0436
	0.0072	0.0285	0.0108	0.0476
minwage bite × 2018	-0.0437***	-0.0737**	-0.0654***	-0.1534***
	0.0076	0.0311	0.012	0.0558
leverage × 2012	-0.0013	-0.0022*	-0.0038*	-0.0039
	0.0012	0.0013	0.002	0.0024
leverage × 2013	0.0001	0	0.0027*	0.0026
	0.001	0.0011	0.0016	0.0019
leverage × 2015	-0.0006	0.0008	0.0031*	0.0083***
	0.0009	0.001	0.0017	0.0021
leverage × 2016	0.0008	0.0015	0.0029	0.0057**
	0.0011	0.0013	0.002	0.0024
leverage × 2017	0.0013	0.0016	0.0064***	0.0091***
	0.0013	0.0015	0.0022	0.0026
leverage × 2018	0.0023	0.0011	0.0062***	0.0050*
	0.0014	0.0016	0.0024	0.0028
minwage bite × leverage 2012	-0.0016	0.0213	-0.0143	-0.0243
	0.0129	0.014	0.022	0.0272
minwage bite × leverage 2013	-0.0107	0.007	-0.0104	-0.0185
	0.0109	0.011	0.0165	0.0192
minwage bite × leverage 2015	-0.0428***	-0.0386***	-0.1033***	-0.1191***
	0.011	0.0128	0.02	0.0241
minwage bite × leverage 2016	-0.0373***	-0.0231	-0.0779***	-0.0969***
	0.0134	0.0148	0.0226	0.027
minwage bite × leverage 2017	-0.0300**	-0.011	-0.0768***	-0.1104***
	0.0145	0.0159	0.0238	0.0283
minwage bite × leverage 2018	-0.0292*	-0.0005	-0.0820***	-0.0628**
	0.015	0.0164	0.026	0.0317
Fixed Effects	Yes	Yes	Yes	Yes
Firm Controls	No	Yes	No	Yes
Observations	1,709,816	1,257,534	1,709,816	1,257,534

Table A.3: Dynamic effect of financial constraints on wage dispersion. The table provides regression coefficients using a dynamic version of Equation (2). The dependent variable is wage dispersion which is defined as the log difference of the 75th (90th) to 25th (10th) wage percentile at the firm level. Minimum wage bite is calculated at the firm level using Equation (1). Leverage is defined as total debt-to-assets. We calculate pre-policy averages at the firm level. Fixed effects include time-invariant firm fixed effects and triple interaction of county-1 digit industry-year fixed effects. Firm controls include triple interactions of pre-policy average total assets, firm age as of 2014, 3-year total assets growth between 2011 and 2014 and firm-level AKM fixed effects with minimum wage bite measure and year fixed effects. Standard errors are clustered at the firm level. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Wage dispersion	log wage p75-p25		log wage p90-p10	
	(1)	(2)	(3)	(4)
minwage bite × post-2014	-0.0898***	-0.0642***	-0.1485***	-0.1055***
	0.0056	0.0041	0.0087	0.0066
financial constraint index	-0.0016	-0.0006	-0.0016	0.0001
	0.0011	0.0006	0.0021	0.0011
minwage bite × financial constraint index × post-2014	-0.0219**	-0.0241***	-0.0256	-0.0342***
	0.0087	0.0067	0.0156	0.0114
Fixed Effects	No	Yes	No	Yes
Observations	1,439,906	1,426,688	1,439,906	1,426,688

Table A.4: Effect of financial constraints on wage dispersion: Robustness with a new financial constraint index. The index takes the value of 1 when the firm is categorized as financially constraint based on the methodology described in the text. The table provides regression coefficients using Equation (2). The dependent variable is wage dispersion which is defined as the log difference of the 75th (90th) to 25th (10th) wage percentile at the firm level. Minimum wage bite is calculated at the firm level using Equation (1). We calculate pre-policy averages at the firm level. Pre (Post) policy time period includes years 2012-2014 (2015-2017). Fixed effects include time-invariant firm fixed effects and triple interaction of county-1 digit industry fixed effects with a policy indicator. Standard errors are clustered at the firm level. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

B Model derivation

B.1 Wage determination

The value functions of employed and unemployed workers are given in equations (13) and (14). We rewrite these equations in steady state as:

$$W_j = w_j + \beta[(1-\delta)W_j + \delta U], \tag{A.1}$$

$$U = b + \beta\Big[f(\theta)\big(1 - H^x(x^*)\big)\bar{W} + \Big(1 - f(\theta)\big(1 - H^x(x^*)\big)\Big)U\Big]. \tag{A.2}$$

The Nash bargaining solution (15) in steady state reads

$$W_i - U = \frac{\eta}{\phi(1-\eta)} J_i \tag{A.3}$$

Rewriting equation (A.1) for W_i, we have:

$$(1-\beta(1-\delta))W_i = w_i + \beta\delta U$$

$$\Rightarrow (1-\beta(1-\delta))(W_i - U) = w_i - (1-\beta)U$$

Substituting for $(W_i - U)$ from equation (A.3), we have:

$$(1-\beta(1-\delta))\frac{\eta}{\phi(1-\eta)} J_i = w_i - (1-\beta)U$$

By substituting for J_i from the steady state version of equation (9), we have:

$$\frac{\eta}{1-\eta}\big(\frac{\Omega}{\phi} y_i - w_i\big) = w_i - (1-\beta)U$$

Therefore, we have:

$$\tilde{w}(y_i) = w_i = \eta\frac{\Omega}{\phi} y_i + (1-\eta)(1-\beta)U \tag{A.4}$$

B.2 Hiring cutoff z^*

The cutoff productivity of hiring can be derived from the steady state version of equation (12):

$$\frac{J^h(z_j^*)}{\Omega_j} = -c_\tau w^m + \frac{\beta}{1-\beta(1-\delta)}(E[y_j|z_j^*] - \frac{\phi_j}{\Omega_j}E[w_j|z_j^*]) = 0, \tag{A.5}$$

Here,

$$E[y_j|z_j^*] = \frac{1}{1-H^\epsilon(x_j^*-z_j^*)}\Big[\int_{\epsilon=x_j^*-z_j^*}^{x_j^m-z_j^*}(z_j^*+\epsilon)dH^\epsilon(\epsilon) + \int_{\epsilon=x_j^m-z_j^*}^{\infty}(z_j^*+\epsilon+e_j^*)dH^\epsilon(\epsilon)dH^\epsilon(\epsilon)\Big] \tag{A.6}$$

or equivalently

$$E[y_j|z_j^*] = \frac{1}{1-H^\epsilon(x_j^*-z_j^*)}\Big[\int_{\epsilon=x_j^*-z_j^*}^{\infty}(z_j^*+\epsilon)dH^\epsilon(\epsilon) + [1-H^\epsilon(x_j^m-z_j^*)]e_j^*\Big] \tag{A.7}$$

and

$$E[w_j|z_j^*] = \frac{1}{1-H^\epsilon(x_j^*-z_j^*)}\Big[\big(H^\epsilon(x_j^m-z_j^*) - H^\epsilon(x_j^*-z_j^*)\big)w^m + \int_{\epsilon=x_j^m-z_j^*}^{\infty}\Big[\tilde{w}(z_j^*+\epsilon+e_j^*)\Big]dH^\epsilon(\epsilon)\Big] \tag{A.8}$$

Here, $\tilde{w}(.)$ is the market wage function. See equation (16).

Consequently, we rewrite equation (A.5) as follows:

$$\begin{aligned} -c_\tau w^m + \frac{\beta}{1-\beta(1-\delta)}\Big[\int_{\epsilon=y_j^*-z_j^*}^{x_j^m-z_j^*}\Big[z_j^*+\epsilon-\frac{\phi_j}{\Omega_j}w^m\Big]dH^\epsilon(\epsilon) \\ + \int_{\epsilon=x_j^m-z_j^*}^{\infty}\Big[(z_j^*+\epsilon+e_j^*) - \frac{\phi_j}{\Omega_j}\tilde{w}(z_j^*+\epsilon+e_j^*)\Big]dH^\epsilon(\epsilon)\Big] = 0 \end{aligned} \tag{A.9}$$

C Comparative statics

C.1 Ceteris paribus effect of an increase in the minimum wage

Result 1. $\frac{\partial y^*}{\partial w^m} > 0$

Proof. This follows directly from equation (19) □

Result 2. $\frac{\partial x^*}{\partial w^m} > 0$ *without effort.* $\frac{\partial x^*}{\partial w^m} < 0$ *only in case of effort and if* $(1-\beta)\frac{dU}{dw^m} < 1$.

Proof. As shown in Section 3.5, $x^* = y^*$ without effort. Hence, in case of no effort $\frac{\partial x^*}{\partial w^m} =$

$\frac{\partial y^*}{\partial w^m} = \frac{\phi_j}{\Omega_j} > 0.$

With effort, $x^* < y^*$. We can then use equation (20) to derive

$$C_e'(y^* - x^*)\frac{d(y^* - x^*)}{dw^m} = 1 - (1-\beta)\frac{dU}{dw^m} \tag{A.10}$$

$$\Rightarrow \frac{d(x^*)}{dw^m} = \frac{\phi_j}{\Omega_j} - \frac{1-(1-\beta)\frac{dU}{dw^m}}{C_e'(y^* - x^*)} \tag{A.11}$$

Note that for $\frac{\partial x^*}{\partial w^m} < 0$ it must hold that $(1-\beta)\frac{dU}{dw^m} < 1$. Hence, the value of unemployment increases only little or possibly falls with an increase in the minimum wage. If $(1-\beta)\frac{dU}{dw^m} > 1$, then $\frac{\partial x^*}{\partial w^m} > 0$. □

Result 3. $\frac{\partial y^m}{\partial w^m} > 0$ *if* $(1-\eta)(1-\beta)\frac{dU}{dw^m} < 1$.

Proof. The cutoff y^m is only meaningful in the case of effort. The derivative of equation (21) w.r.t. w^m then implies:

$$\frac{dy^m}{dw^m} = \frac{1}{\eta}\frac{\phi}{\Omega}[1 - (1-\eta)(1-\beta)\frac{dU}{dw^m}] \tag{A.12}$$

If $(1-\eta)(1-\beta)\frac{dU}{dw^m} < 1$, $\frac{\partial y^m}{\partial w^m} > 0$. □

Result 4. $\frac{dx^m}{dw^m} = \frac{dy^m}{dw^m}$

Proof. The cutoff x^m is only meaningful in the case of effort. The derivative of equation (22) w.r.t. w^m implies:

$$\eta\frac{\Omega}{\phi}(\frac{dx^m}{dw^m} + \frac{de^*}{dw^m} - \frac{dy^m}{dw^m}) = C'(e^*)\frac{de^*}{dw^m}$$

Since $\frac{de^*}{dw^m} = 0$, the above equation becomes:

$$\eta\frac{\Omega}{\phi}(\frac{dx^m}{dw^m} - \frac{dy^m}{dw^m}) = 0 \tag{A.13}$$

Therefore, $\frac{dx^m}{dw^m} = \frac{dy^m}{dw^m}$ □

Result 5. $\frac{\partial z^*}{\partial w^m} > 0$ *without effort.*

Proof. We can rewrite equation (12) in steady state as

$$-c_\tau w^m + \frac{\beta}{1-\beta(1-\delta)} \Big[\int_{\epsilon=y_j^*-z_j^*}^{x_j^m-z_j^*} \Big[z_j^* + \epsilon - \frac{\phi_j}{\Omega_j} w^m \Big] dH^\epsilon(\epsilon) \\ + \int_{\epsilon=x_j^m-z_j^*}^{\infty} \Big[(z_j^* + \epsilon + e_j^*) - \frac{\phi_j}{\Omega_j} \tilde{w}(z_j^* + \epsilon + e_j^*) \Big] dH^\epsilon(\epsilon) \Big] = 0. \tag{A.14}$$

The total derivative of equation (A.14) w.r.t. w^m is:

$$-c_\tau - \frac{\beta}{1-\beta(1-\delta)} \frac{\phi}{\Omega} [H^\epsilon(x^m - z^*) - H^\epsilon(y^* - z^*)] \\ - \frac{\beta}{1-\beta(1-\delta)} \big(e^* - \frac{\phi}{\Omega} C_e(e^*)\big) \frac{dx^m}{dw^m} \\ + \frac{\beta}{1-\beta(1-\delta)} \frac{\partial \Psi}{\partial z^*} \frac{dz^*}{dw^m} = 0 \tag{A.15}$$

The first and second term of this equation (first line) are unambiguously negative.

In the last term of equation (A.15), Ψ is defined as as follows:

$$\Psi := E[y_j|z^*] - \frac{\phi}{\Omega} E[w_j|z^*]$$

which can be rewritten as:

$$\Psi := \int_{\epsilon_j=y^*-z^*}^{x^m-z^*} \Big[z^* + \epsilon_j - \frac{\phi}{\Omega} w^m \Big] dH^\epsilon(\epsilon) + \int_{\epsilon_j=x^m-z^*}^{\infty} \Big[(z^* + \epsilon_j + e^*) - \frac{\phi}{\Omega} \tilde{w}(z^* + \epsilon_j + e^*) \Big] dH^\epsilon(\epsilon)$$

We now derive $\frac{\partial \Psi}{\partial z^*} > 0$.

$$\frac{\partial \Psi}{\partial z^*} = - \left[x^m - \frac{\phi}{\Omega} w^m\right] + \left[x^m + e^* - \frac{\phi}{\Omega} \tilde{w}(x^m + e^*)\right] \\ + [H^\epsilon(x^m - z^*) - H^\epsilon(x^m - y^*)] + [(1 - H^\epsilon(x^m - z^*))(1 - \eta)]$$

Here, $\frac{\partial \tilde{w}(z^*+\epsilon_j+e^*)}{\partial z^*} = \eta \frac{\Omega}{\phi}$ is used to derive the last term in the above equation. From equation (22), we know that $\tilde{w}(x^m + e^*) - w^m = C_e(e^*)$. Substituting this in the equation, we get:

$$\frac{\partial \Psi}{\partial z^*} = \left[e^* - \frac{\phi}{\Omega} C_e(e^*)\right] + [H^\epsilon(x^m - z^*) - H^\epsilon(x^m - y^*)] + [(1 - H^\epsilon(x^m - z^*))(1 - \eta)] \tag{A.16}$$

The second and third term on the RHS of equation (A.16) are both positive. The first term is also positive, since $e^* - \frac{\phi}{\Omega} C_e(e^*) > 0$.

In the case of no effort, the third term in equation (A.15) is zero. This then means that the last term of equation (A.15) must be positive. Since $\frac{\partial \Psi}{\partial z^*} > 0$, we then have $\frac{dz^*}{dw^m} > 0$.

With effort, remember that $C'_e(e^*) = \eta\frac{\Omega}{\phi}$. Since $C_e(0) = 0$ and $C''_e(.) > 0$, we obtain $C_e(e^*) < \eta\frac{\Omega}{\phi}e^*$. Therefore, as $\eta \leq 1$, we conclude that $C_e(e^*) < \frac{\Omega}{\phi}e^* \Leftrightarrow e^* - \frac{\phi}{\Omega}C_e(e^*) > 0$. The term $\frac{dx^m}{dw^m} > 0$ if $(1-\eta)(1-\beta)\frac{dU}{dw^m} < 1$ as we show in Results 3 and 4. In this case, the third term of equation (A.15) is therefore negative in case of effort and $\frac{dz^*}{dw^m} > 0$. Otherwise, the sign of $\frac{dz^*}{dw^m}$ is ambiguous.

□

Result 6. $\frac{\partial v}{\partial w^m} < 0$ *without effort. With effort,* $\frac{\partial v}{\partial w^m} < 0$ *only if* $(1-\beta)\frac{dU}{dw^m} < 1$.

Proof. We can rewrite equation (11) as follows:

$$\begin{aligned}\frac{C'_v(v)}{p(\theta)} =& \frac{\beta}{1-\beta(1-\delta)}\Big[\int_{z_i=z^*}^{\infty}\int_{\epsilon=y^*-z_i}^{x^m-z_i}\big[z_i+\epsilon-\frac{\phi}{\Omega}w^m\big]dH^\epsilon(\epsilon)dH^z(z_i)\\ &+\int_{z_i=z^*}^{\infty}\int_{\epsilon=x^m-z_i}^{\infty}\big[(z_i+e^*+\epsilon)-\frac{\phi}{\Omega}\tilde{w}(z_i+e^*+\epsilon)\big]dH^\epsilon(\epsilon)dH^z(z_i)\Big]-(1-H^z(z^*))c_\tau w^m\end{aligned} \tag{A.17}$$

Let us denote the right-hand side (RHS) of this equation by Υ. The total derivative of this equation w.r.t. w^m is:

$$\frac{d(\frac{C'_v(v)}{p(\theta)})}{dv}\frac{dv}{dw^m} = \frac{\partial\Upsilon}{\partial z^*}\frac{\partial z^*}{\partial w^m} + \frac{\partial\Upsilon}{\partial x^m}\frac{\partial x^m}{\partial w^m} + \frac{\partial\Upsilon}{\partial y^*}\frac{\partial y^*}{\partial w^m} + \frac{\partial\Upsilon}{\partial w^m} \tag{A.18}$$

Note that $\frac{d(\frac{C'_v(v)}{p(\theta)})}{dv} > 0$. Therefore, the statement is proven if we show that the RHS is negative. We compute each term of the RHS separately:

$$\begin{aligned}\frac{\partial z^*}{\partial w^m}\frac{\partial\Upsilon}{\partial z^*} =& \frac{\partial z^*}{\partial w^m}h^z(z^*)\Big[c_\tau w^m - \frac{\beta}{1-\beta(1-\delta)}\Big(\int_{\epsilon_j=y^*-z^*}^{x^m-z^*}\big[z^*+\epsilon_j-\frac{\phi}{\Omega}w^m\big]dH^\epsilon(\epsilon)\\ &+\int_{\epsilon_j=x^m-z^*}^{\infty}\big[(z^*+\epsilon_j+e^*)-\frac{\phi}{\Omega}\tilde{w}(z^*+\epsilon_j+e^*)\big]dH^\epsilon(\epsilon)\Big)\Big]\end{aligned} \tag{A.19}$$

The term inside the square brackets is the LHS of equation (A.9), which is equal to zero. Now we calculate the second term on the RHS of equation (A.18):

$$\frac{\partial x^m}{\partial w^m}\frac{\partial \Upsilon}{\partial x^m} = \frac{\partial x^m}{\partial w^m}\frac{\beta}{1-\beta(1-\delta)}\Big[\int_{z_i=z^*}^{\infty}(x^m - \frac{\phi}{\Omega}w^m)dH^z(z_i) - \int_{z_i=z^*}^{\infty}(x^m + e^* - \frac{\phi}{\Omega}\tilde{w}(x^m+e^*))dH^z(z_i)\Big] \tag{A.20}$$

$$= \frac{\partial x^m}{\partial w^m}\frac{\beta}{1-\beta(1-\delta)}\Big[\int_{z_i=z^*}^{\infty}[x^m - \frac{\phi}{\Omega}w^m - (x^m + e^* - \frac{\phi}{\Omega}\tilde{w}(x^m+e^*))]dH^z(z_i)\Big] \tag{A.21}$$

$$= -\frac{\partial x^m}{\partial w^m}\frac{\beta}{1-\beta(1-\delta)}\Big[\int_{z_i=z^*}^{\infty}[e^* + \frac{\phi}{\Omega}(w^m - \tilde{w}(x^m+e^*))]dH^z(z_i)\Big] \tag{A.22}$$

$$= -\frac{\partial x^m}{\partial w^m}\frac{\beta}{1-\beta(1-\delta)}\Big[\int_{z_i=z^*}^{\infty}[e^* - \frac{\phi}{\Omega}C_e(e^*)]dH^z(z_i)\Big] \tag{A.23}$$

$$= -\frac{\partial x^m}{\partial w^m}\frac{\beta}{1-\beta(1-\delta)}[e^* - \frac{\phi}{\Omega}C_e(e^*)]\,(1-H^z(z^*)) \tag{A.24}$$

Note that in the last step, we use equation (22). Since $e^* - \frac{\phi}{\Omega}C_e(e^*) > 0$, $\frac{\partial x^m}{\partial w^m}\frac{\partial \Upsilon}{\partial x^m} < 0$.

Now we calculate the third term on the RHS of equation (A.18):

$$\frac{\partial y^*}{\partial w^m}\frac{\partial \Upsilon}{\partial z^y} = \frac{\partial y^*}{\partial w^m}\frac{\beta}{1-\beta(1-\delta)}\Big(\int_{z_i=z^*}^{\infty}\overbrace{[y^* - \frac{\phi}{\Omega}w^m]}^{=0}\,dH^z(z_i)\Big) = 0 \tag{A.25}$$

Finally the last term:

$$\begin{aligned}\frac{\partial \Upsilon}{\partial w^m} =& \frac{\beta}{1-\beta(1-\delta)}\Big[\int_{z_i=z^*}^{\infty}\int_{\epsilon=y^*-z_i}^{x^m-z_i}(-\frac{\phi}{\Omega})\Big]dH^{\epsilon}(\epsilon)dH^z(z_i) - (1-H^z(z^*))c_{\tau} \\ =& -\frac{\beta}{1-\beta(1-\delta)}\big(H^x(x^m) - H^x(y^*)\big)\frac{\phi}{\Omega}\big) - (1-H^z(z^*))c_{\tau} < 0\end{aligned}$$

Therefore, in equation (A.18), we have:

$$\begin{aligned}\frac{\overbrace{d(\frac{C_v'(v)}{p(\theta)})}^{>0}}{dv}\frac{dv}{dw^m} &= \overbrace{\frac{\partial \Upsilon}{\partial z^*}}^{=0}\frac{\partial z^*}{\partial w^m} + \overbrace{\frac{\partial \Upsilon}{\partial x^m}}^{<0}\overbrace{\frac{\partial x^m}{\partial w^m}}^{>0} + \overbrace{\frac{\partial \Upsilon}{\partial y^*}}^{=0}\frac{\partial y^*}{\partial w^m} + \overbrace{\frac{\partial \Upsilon}{\partial w^m}}^{<0} \\ &= \overbrace{\frac{\partial \Upsilon}{\partial x^m}}^{<0}\overbrace{\frac{\partial x^m}{\partial w^m}}^{>0} + \overbrace{\frac{\partial \Upsilon}{\partial w^m}}^{<0}\end{aligned}$$

Hence, $\frac{dv}{dw^m} < 0$ without effort, since in this case $\frac{\partial x^m}{\partial w^m} = 0$. With effort, this result still holds if $(1-\beta)\frac{dU}{dw^m} < 1$.

□

Result 7. $\frac{\partial n}{\partial w^m} < 0$ *without effort. With effort,* $\frac{\partial n}{\partial w^m} > 0$ *only if* $(1-\beta)\frac{dU}{dw^m} < 1$.

Proof. We rewrite steady state employment as

$$n_j = \frac{1}{\delta} M(v_j, u)\big(1 - H_j^x(x_j^*)\big). \tag{A.26}$$

We know that $\frac{\partial M(v_j,u)}{\partial v_j} > 0$. Without effort, we have shown that $\frac{\partial v}{\partial w^m} < 0$ and that $\frac{\partial x^*}{\partial w^m} > 0$. Hence, $\frac{\partial n}{\partial w^m} < 0$ follows from the above equation.

With effort, we have shown that $\frac{\partial v}{\partial w^m} < 0$ and that $\frac{\partial x^*}{\partial w^m} < 0$ if $(1-\beta)\frac{dU}{dw^m} < 1$. Hence, $\frac{\partial n}{\partial w^m} > 0$ follows from the above equation. If $(1-\beta)\frac{dU}{dw^m} > 1$, the employment effect of the minimum wage is ambiguous in the case of effort. □

C.2 Ceteris paribus effect of an increase in financial constraints

Result 8. $\frac{d\phi}{d\mu} > 0$

Proof. Here, we use the Envelope theorem. First, we show that $\Omega < \phi$:

$$\Omega < \phi \Leftrightarrow 1 - \Gamma(\bar{\omega}) + \phi[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] < \phi \Leftrightarrow 1 - \Gamma(\bar{\omega}) < \phi[1 - \Gamma(\bar{\omega}) + \mu G(\bar{\omega})] \Leftrightarrow \phi > \frac{1 - \Gamma(\bar{\omega})}{1 - \Gamma(\bar{\omega}) + \mu G(\bar{\omega})}$$

The RHS of the last inequality is strictly smaller than 1, while ϕ is strictly larger than 1. Therefore, $\Omega < \phi$. □

Result 9. $\frac{d(\frac{\phi}{\Omega})}{d\mu} > 0$

Proof. To find $\frac{d(\frac{\phi}{\Omega})}{d\mu}$, first we derive $\frac{d\Omega}{d\mu}$:

$$\begin{aligned}
&\frac{d\Omega}{d\mu} = \frac{\partial\Omega}{\partial\mu} + \frac{\partial\Omega}{\partial\phi}\frac{d\phi}{d\mu} + \frac{\partial\Omega}{\partial\bar{\omega}}\frac{\partial\bar{\omega}}{\partial\mu} \\
\Rightarrow &\frac{d\Omega}{d\mu} = -G\phi + [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]\frac{d\phi}{d\mu} + \overbrace{\big[1 - \Gamma'(\bar{\omega}) + \phi\big(\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})\big)\big]}^{=0}\frac{\partial\bar{\omega}}{\partial\mu} \\
\Rightarrow &\frac{d\Omega}{d\mu} = -G(\bar{\omega})\phi + [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]\frac{d\phi}{d\mu}
\end{aligned}$$

Now we derive $\frac{d(\frac{\phi}{\Omega})}{d\mu}$:

$$\begin{aligned}
\frac{d(\frac{\phi}{\Omega})}{d\mu} &= \frac{d(\frac{\lambda^w\Omega+(1-\lambda^k)\phi}{\Omega})}{d\mu} \\
\Rightarrow \frac{d(\frac{\phi}{\Omega})}{d\mu} &= \frac{(1-\lambda^w)\left[\frac{d\phi}{d\mu}\Omega - \frac{d\Omega}{d\mu}\phi\right]}{\Omega^2} \\
\Rightarrow \frac{d(\frac{\phi}{\Omega})}{d\mu} &= \frac{(1-\lambda^w)\left[\frac{d\phi}{d\mu}\Omega + G(\bar{\omega})\phi^2 - \phi[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]\frac{d\phi}{d\mu}\right]}{\Omega^2} \\
\Rightarrow \frac{d(\frac{\phi}{\Omega})}{d\mu} &= \frac{(1-\lambda^w)\left[\left(1-\Gamma(\bar{\omega})\right)\frac{d\phi}{d\mu} + G(\bar{\omega})\phi^2\right]}{\Omega^2}
\end{aligned}$$

Since $1-\Gamma(\bar{\omega}) > 0$ and $\frac{d\phi}{d\mu} > 0$, we have $\frac{d(\frac{\phi}{\Omega})}{d\mu} > 0$.

□

Result 10. $\frac{dx^*}{d\mu} = \frac{dy^*}{d\mu} > 0$

Proof. Taking the derivative of equation (19) w.r.t. μ yields

$$\frac{dy^*}{d\mu} = w^m \frac{\partial(\frac{\phi}{\Omega})}{\partial\mu} > 0 \tag{A.27}$$

In equation (20), w^m and U are independent of μ. Consequently, $y^* - x^*$ does not change with μ which implies

$$\frac{dx^*}{d\mu} = \frac{dy^*}{d\mu} > 0. \tag{A.28}$$

□

Result 11. $\frac{dy^m}{d\mu} > 0$

Proof. Taking the derivative of equation (21) w.r.t. μ yields

$$\frac{dy^m}{d\mu}\frac{\Omega}{\phi} + \frac{\partial(\frac{\Omega}{\phi})}{\partial\mu} y^m = 0 \tag{A.29}$$

Since $\frac{\partial(\frac{\Omega}{\phi})}{\partial\mu} < 0$, it follows that $\frac{dy^m}{d\mu} > 0$. □

Result 12. $\frac{dx^m}{d\mu} > \frac{dy^m}{d\mu} > 0$

Proof. The derivative of equation (22) w.r.t. μ is given by

$$\eta\frac{\partial(\frac{\Omega}{\phi})}{\partial\mu}(x^m+e^*-y^m)+\eta\frac{\Omega}{\phi}\Big[\frac{dx^m}{d\mu}+\frac{de^*}{d\mu}-\frac{dy^m}{d\mu}\Big]=C_e'(e^*)\frac{de^*}{d\mu} \tag{A.30}$$

$$\Rightarrow\frac{\Omega}{\phi}\Big[\frac{dx^m}{d\mu}-\frac{dy^m}{d\mu}\Big]=-\frac{\partial(\frac{\Omega}{\phi})}{\partial\mu}(x^m+e^*-y^m)+\big[C_e'(e^*)-\eta\frac{\Omega}{\phi}\big]\frac{de^*}{d\mu} \tag{A.31}$$

From equation (23), we know that the second term on the RHS of this equation is zero. Since $\frac{\partial(\frac{\Omega}{\phi})}{\partial\mu}<0$, it follows that $\frac{dx^m}{d\mu}>\frac{dy^m}{d\mu}$. □

Result 13. $\frac{dz^*}{d\mu}>0$

Proof. The total derivative of equation (A.14) w.r.t. μ is:

$$\frac{\partial\Psi}{\partial z^*}\frac{dz^*}{d\mu}+\frac{\partial\Psi}{\partial y^*}\frac{dy^*}{d\mu}+\frac{\partial\Psi}{\partial x^m}\frac{dx^m}{d\mu}+\frac{\partial\Psi}{\partial e^*}\frac{de^*}{d\mu}+\frac{\partial\Psi}{\partial(\frac{\phi}{\Omega})}\frac{d(\frac{\phi}{\Omega})}{d\mu}=0 \tag{A.32}$$

where Ψ is defined in Result 5 and it is shown that $\frac{\partial\Psi}{\partial z^*}>0$. Using equation (A.14), We can rewrite equation (A.32) as:

$$\frac{\partial\Psi}{\partial z^*}\frac{dz^*}{d\mu}-\frac{dy^*}{d\mu}\Big[\overbrace{y^*-\frac{\phi}{\Omega}w^m}^{=0}\Big]-\frac{dx^m}{d\mu}\Big[\big(x^m+e^*-\frac{\phi}{\Omega}\tilde{w}(x^m+e^*)\big)-(x^m-w^m)\Big]$$
$$-\frac{d(\frac{\phi}{\Omega})}{d\mu}\Big[\int_{\epsilon_j=y^*-z^*}^{x^m-z^*}w^m dH^\epsilon(\epsilon)+\int_{\epsilon_j=x^m-z^*}^{\infty}\tilde{w}(x^m+e^*)dH^\epsilon(\epsilon)\Big]+\frac{de^*}{d\mu}\int_{\epsilon_j=x^m-z^*}^{\infty}\overbrace{[1-\frac{\phi}{\Omega}\frac{\Omega}{\phi}]}^{=0}=0$$

The proof for Result 5 also shows that $\big(x^m+e^*-\frac{\phi}{\Omega}\tilde{w}(x^m+e^*)\big)-(x^m-w^m)=e^*-\frac{\phi}{\Omega}C_e(e^*)>0$. Therefore:

$$\frac{\partial\Psi}{\partial z^*}\frac{dz^*}{d\mu}-\frac{dx^m}{d\mu}\Big[\big(e^*-\frac{\phi}{\Omega}C_e(e^*)\big)\Big]$$
$$-\frac{d(\frac{\phi}{\Omega})}{d\mu}\Big[\int_{\epsilon_j=y^*-z^*}^{x^m-z^*}w^m dH^\epsilon(\epsilon)+\int_{\epsilon_j=x^m-z^*}^{\infty}\tilde{w}(x^m+e^*)dH^\epsilon(\epsilon)\Big]=0$$

The second term of this equation represents the firm's expected loss through the increase in x^m, as a lower share of workers would exert effort e^*, but the firm also pays lower wages to this group. The last term indicates that higher μ increases the effective cost of wage payments. Since both terms are negative, $\frac{\partial\Psi}{\partial z^*}\frac{dz^*}{d\mu}$ must be strictly positive. As $\frac{\partial\Psi}{\partial z^*}>0$ (see Result 5), we result that $\frac{dz^*}{d\mu}>0$.

□

Result 14. $\frac{dv}{d\mu} < 0$

Proof. Remember from Result 6 that we can rewrite job creation condition (11) as follows:

$$\frac{C'_v(v)}{p(\theta)} = \Upsilon \tag{A.33}$$

where Υ is defined in Result 6. The total derivative of this equation w.r.t. μ is:

$$\frac{d(\frac{C'_v(v)}{p(\theta)})}{dv}\frac{dv}{d\mu} = \frac{\partial\Upsilon}{\partial z^*}\frac{dz^*}{d\mu} + \frac{\partial\Upsilon}{\partial x^m}\frac{dx^m}{d\mu} + \frac{\partial\Upsilon}{\partial y^*}\frac{dy^*}{d\mu} + \frac{\partial\Upsilon}{\partial e^*}\frac{de^*}{d\mu} + \frac{\partial\Upsilon}{\partial(\frac{\phi}{\Omega})}\frac{d(\frac{\phi}{\Omega})}{d\mu}$$

Remember that $\frac{d(\frac{C'_v(v)}{p(\theta)})}{dv} > 0$. Therefore, the statement is proven if we show that the RHS is negative. Result 6 shows that $\frac{\partial\Upsilon}{\partial x^m} < \frac{\partial\Upsilon}{\partial z^*} = \frac{\partial\Upsilon}{\partial y^*} = 0$.

Also, note that $\frac{\partial\Upsilon}{\partial e^*} = \int_{z=z^*}^{\infty}\int_{\epsilon=x^m-z}^{\infty}[1 - \overbrace{\frac{\phi}{\Omega}\frac{\Omega}{\phi}}^{=0}]dH^\epsilon(\epsilon)dH^z(z) = 0$. Moreover, $\frac{\partial\Upsilon}{\partial(\frac{\phi}{\Omega})} = 0$ which can be directly observed by making a partial derivative of Υ w.r.t. $\frac{\phi}{\Omega}$. From Result 12, we have $\frac{dx^m}{d\mu} > 0$ which together with Result 6 determines $\frac{dv}{d\mu} < 0$. □

C.3 Interaction of minimum wage and financial constraints

Result 15. $\frac{d^2y^*}{dw^m \cdot d\mu} > 0$

Proof. Taking the derivative of equation (19) with respect to w^m and then with respect to μ yields

$$\frac{d^2y^*}{dw^m \cdot d\mu} = \frac{d(\frac{\phi}{\Omega})}{d\mu} > 0 \tag{A.34}$$

□

Result 16. $\frac{d^2x^*}{dw^m \cdot d\mu} = \frac{d^2y^*}{dw^m \cdot d\mu} > 0$

Proof. Taking the derivative of equation (A.11) with respect to μ gives

$$C''_e(y^* - x^*)\frac{d(y^* - x^*)}{d\mu}\frac{d(y^* - x^*)}{dw^m} + C'_e(y^* - x^*)\frac{d^2(y^* - x^*)}{dw^m \cdot d\mu} = 0 \tag{A.35}$$

From equation (A.28), we know that $\frac{d(y^*-x^*)}{d\mu} = 0$. Therefore:

$$\frac{d^2(y^* - x^*)}{dw^m \cdot d\mu} = 0 \tag{A.36}$$

$$\Rightarrow \frac{d^2(x^*)}{dw^m \cdot d\mu} = \frac{d^2(y^*)}{dw^m \cdot d\mu} \tag{A.37}$$

□

Result 17. $\frac{d^2y^m}{dw^m \cdot d\mu} > 0$ *if* $(1-\beta)\frac{dU}{dw^m} < 1$

Proof. Taking the derivative of equation (A.12) with respect to μ gives

$$\frac{d^2y^m}{dw^m \cdot \mu} = \frac{1}{\eta}\frac{d(\frac{\phi}{\Omega})}{d\mu}[1 - (1-\eta)(1-\beta)\frac{dU}{dw^m}] > 0 \tag{A.38}$$

□

Result 18. $\frac{d^2x^m}{dw^m \cdot d\mu} = \frac{d^2y^m}{dw^m \cdot d\mu} > 0$

Proof. Taking the derivative of equation (A.13) with respect to μ yields

$$\eta\frac{d(\frac{\Omega}{\phi})}{d\mu}(\frac{dx^m}{dw^m} - \frac{dy^m}{dw^m}) + \eta\frac{\Omega}{\phi}(\frac{d^2x^m}{dw^m \cdot d\mu} - \frac{d^2y^m}{dw^m \cdot d\mu}) = 0$$

From equation (A.13), we know that $\frac{dx^m}{dw^m} - \frac{dy^m}{dw^m} = 0$. Therefore

$$\frac{d^2x^m}{dw^m \cdot d\mu} = \frac{d^2y^m}{dw^m \cdot d\mu} \tag{A.39}$$

□

D Additional quantitative results

	No minimum wage ($w^m = 0$)		Binding min. wage ($w^m = 1.48 > 0$)	
Variable	Base	Financial frictions	Base	Financial frictions
Employment	0.940	0.556	0.935	0.530
Vacancies	0.056	0.042	0.055	0.040
Av. wage	1.462	1.455	1.500	1.494
Av. productivity	1.902	2.005	1.927	2.036
Av. output (per worker)	1.904	2.006	1.929	2.038
w^m takers (share)	-	-	0.436	0.499
Firing rate	0.042	0.059	0.042	0.061
Premium	0	0.068	0	0.070
$log(w_{75}) - log(w_{25})$	0.03596	0.03101	0.02436	0.01762
Δ			*-0.01161*	*-0.01338*
$log(w_{90}) - log(w_{10})$	0.06620	0.05738	0.04612	0.03677
Δ			*-0.02008*	*-0.02061*

Table A.5: Steady state comparison across different models with high effort costs. Aggregate variables marked with * are kept constant when investigating the effect of financial constraints in a single firm ($\mu > 0$).

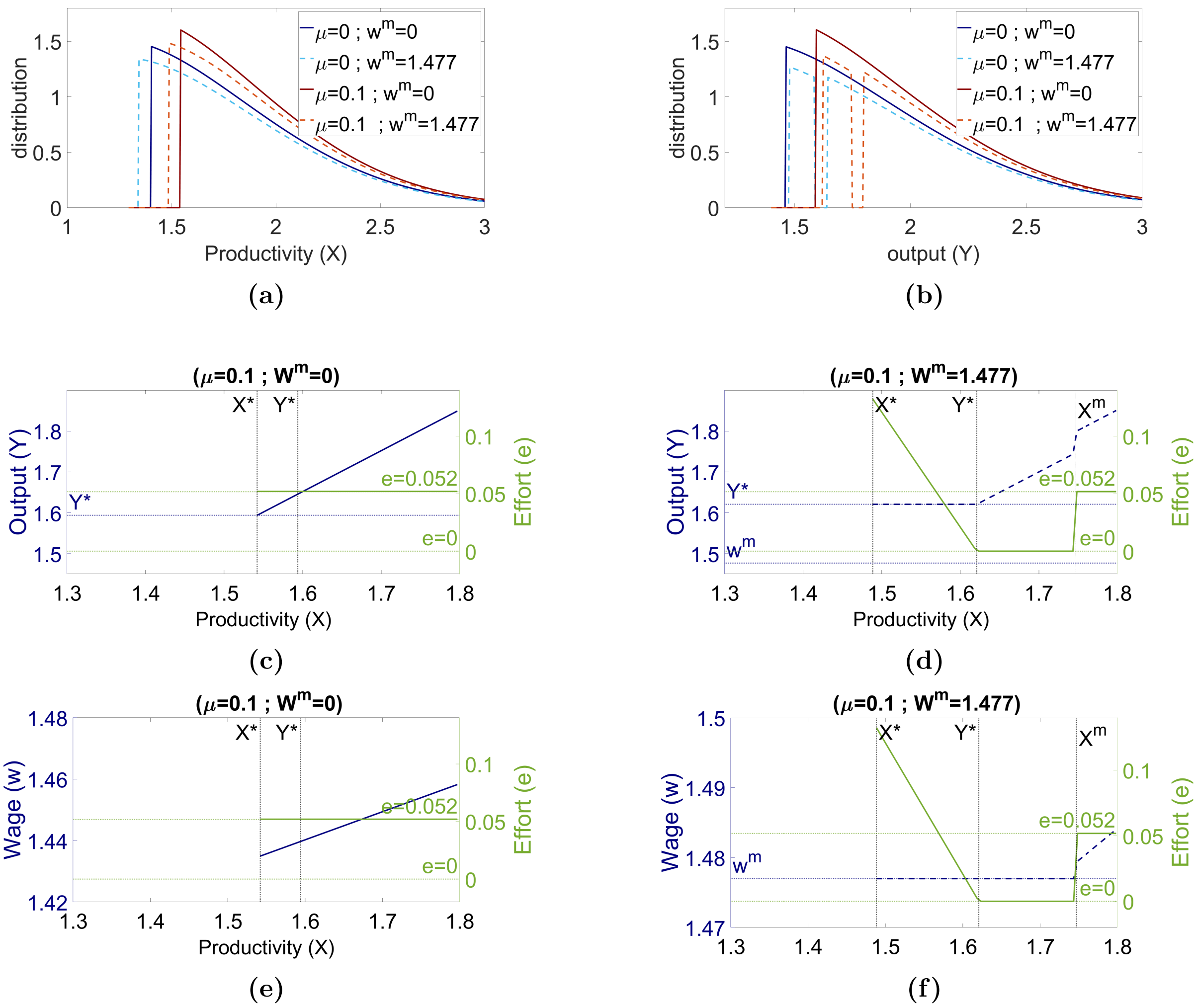

Figure A.1: Distribution of productivity and output, effort and wages conditional on productivity in the counterfactual economies without ($w^m = 0$) and with a minimum wage ($w^m > 0$); with financial constraints ($\mu > 0$).

	(1) Medium effort	(2) Low effort	(3) High effort	(4) Productivity target
Panel a: Base (no financial frictions)				
Employment	0.00	-0.32	0.32	0.00
Vacancies posted	-1.15	0.28	-2.84	-1.18
Hiring	-0.86	1.94	-4.01	-2.91
Firing	-1.11	2.49	-5.16	-3.59
Av. wage	0.38	1.15	-0.67	0.12
Wage dispersion (90-10)	-1.61	-2.78	0.63	-0.67
Wage dispersion (75-25)	0.01	-0.46	0.58	0.07
Av. output (per worker)	-0.12	1.16	-1.76	-0.17
Finance premium	-	-	-	-
Panel b: Financial frictions (one firm)				
Employment	-0.68	-0.57	-0.91	-1.09
Vacancies posted	-1.52	0.23	-3.55	-1.73
Hiring	-1.22	1.78	-4.81	-3.81
Firing	-1.42	2.41	-5.87	-4.55
Av. wage	0.38	1.24	-0.82	0.12
Wage dispersion (90-10)	-1.83	-2.88	0.23	-0.78
Wage dispersion (75-25)	-0.06	-0.47	0.45	0.03
Av. output (per worker)	0.00	1.21	-1.51	-0.03
Finance premium	0.03	0.00	0.10	0.02
Panel c: Financial frictions (market)				
Employment	-0.22	-0.32	0.11	-0.22
Vacancies posted	-1.29	0.46	-3.44	-1.24
Hiring	0.11	3.01	-2.49	-2.78
Firing	0.28	4.00	-3.20	-3.55
Av. wage	0.85	1.80	0.08	0.60
Wage dispersion (90-10)	-3.12	-4.19	-0.68	-0.78
Wage dispersion (75-25)	-0.29	-0.72	0.03	0.03
Av. output (per worker)	0.48	1.77	-0.88	0.44
Finance premium	0.10	0.00	0.10	0.00

Table A.6: Steady state change when introducing a minimum wage across counterfactual models (in percent, except for finance premium and wage dispersion in percentage points).